Tunable Proximity Effects and Topological Superconductivity in Semiconductor-Superconductor-Ferromagnetic Hybrid Nanowires

Samuel D. Escribano

Theoretical Condensed Matter Physics Department
Universidad Autónoma de Madrid

Introduction

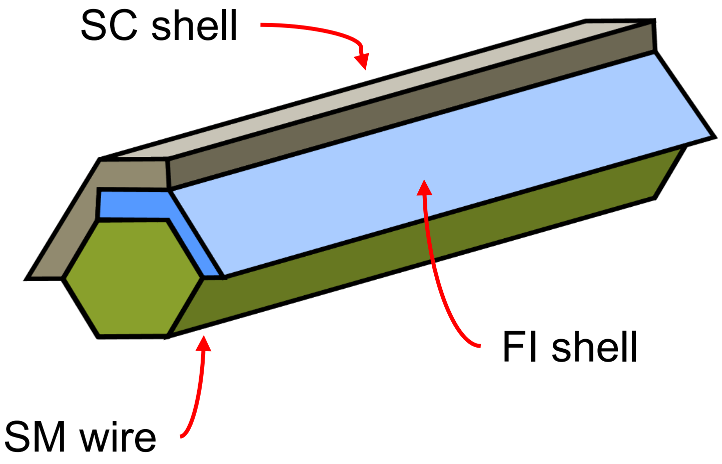
Arxiv preprint:
Microscopic analysis of topological superconductivity in ferromagnetic hybrid nanowires
arXiv:2011.06566

Alfredo Levy Yeyati
UAM

Elsa Prada
ICMM-CSIC

Yuval Oreg
Weizmann Institute

Hybrid nanowire



Applications in:
- Spintronics
- **Topological superconductivity**

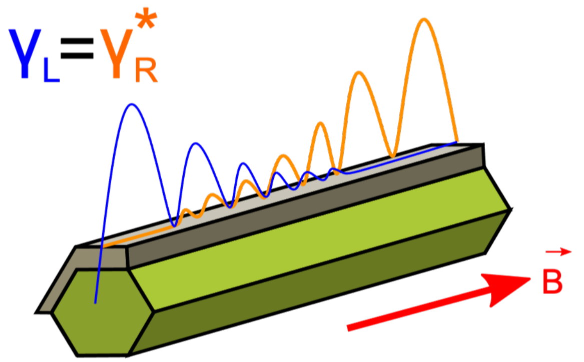
Introduction

- MBS are topological subgap modes that can emerge at the ends of a superconductor/semiconductor nanowire

$$H = \left(\frac{\hbar^2 k^2}{2m^*} - \mu + V_Z \sigma_x + \vec{\alpha} \cdot (\vec{\sigma} \times \vec{k})\right) \tau_z - i\Delta \sigma_y \tau_y$$

Ingredients: Kinetic energy | Electro-chemical potential | SO interaction | Induced superconductivity

Zeeman field: $V_Z = \frac{1}{2}\mu_B g B_x$



$V_Z \geq \sqrt{\Delta^2 + \mu^2}$

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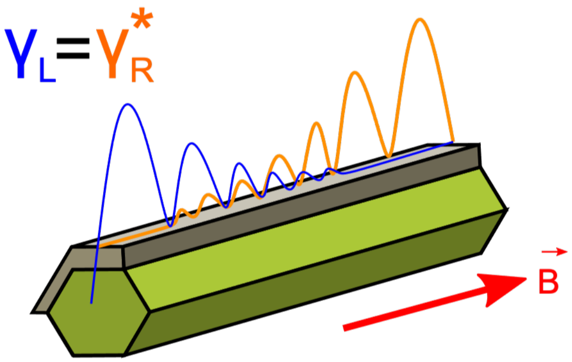

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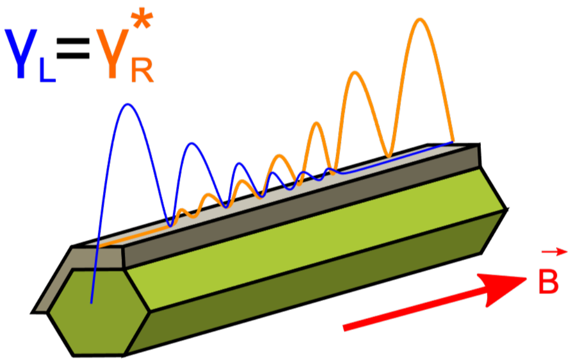

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But the magnetic field **weakens the superconductivity** and **complicates the scaling of a QC**...

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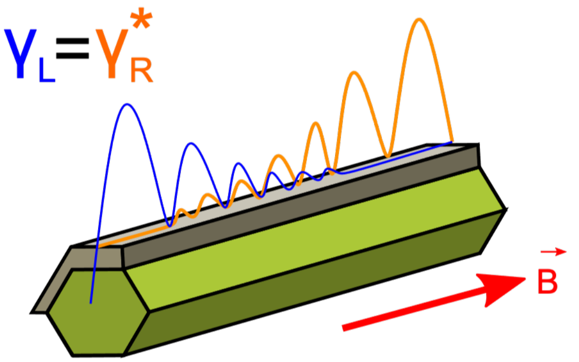

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- MBS may be useful for (topological) quantum computing.

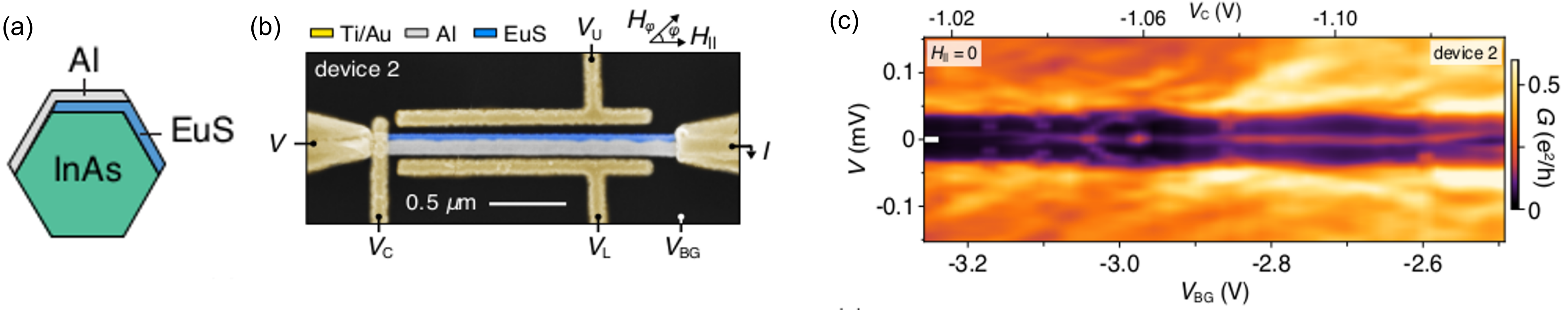
But the magnetic field **weakens the superconductivity** and **complicates the scaling of a QC**...

Is it possible to create MBS without a magnetic field?

Introduction

There is no need of an external magnetic field if it can be intrinsically incorporated. Recent experimental works show that it is possible to induce an exchange field in the nanowire by proximitizing an EuS layer to the heterostructure.

This device shows ZBP compatible with the existence of MBS.



Refs. Y. Liu *et al.*, ACS App. Mat. **12**, 8780 (2020)
Y. Liu *et al.*, Nano Lett. **20**, 456 (2020)
S. Vaitiekėnas *et al.* Nat. Phys. 17, 43 (2021)

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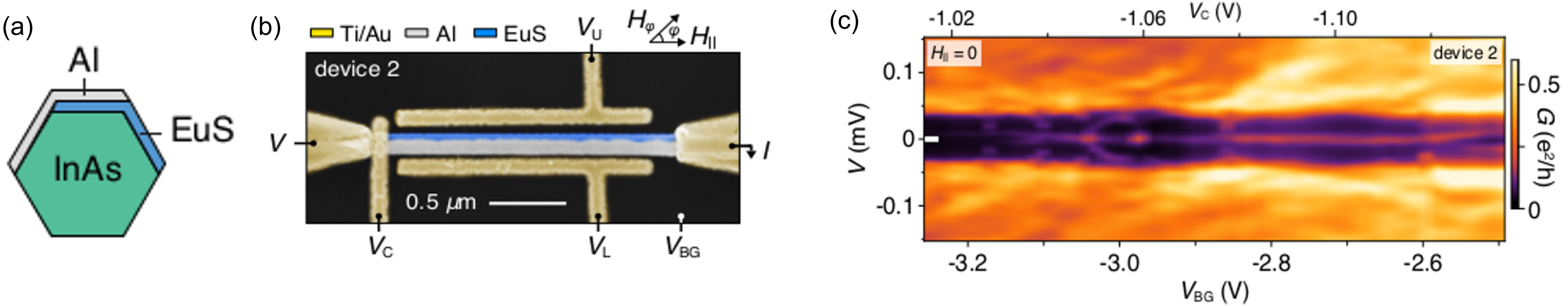
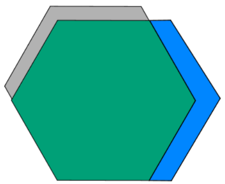


Strikingly, other geometries show little or no induced magnetization



Could be the ZBP Majorana Bound States? Why these devices do not show ZBP? How is induced the magnetization?

Introduction

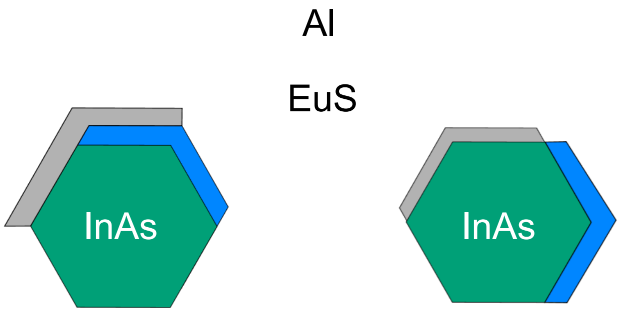
- Realistic model
    + Overlapping device
    + Non-overlapping device

Useful to understand the induced magnetization

- Effective model
    + Overlapping device
    + Non-overlapping device

Useful to study the phase diagram

- Conclusions



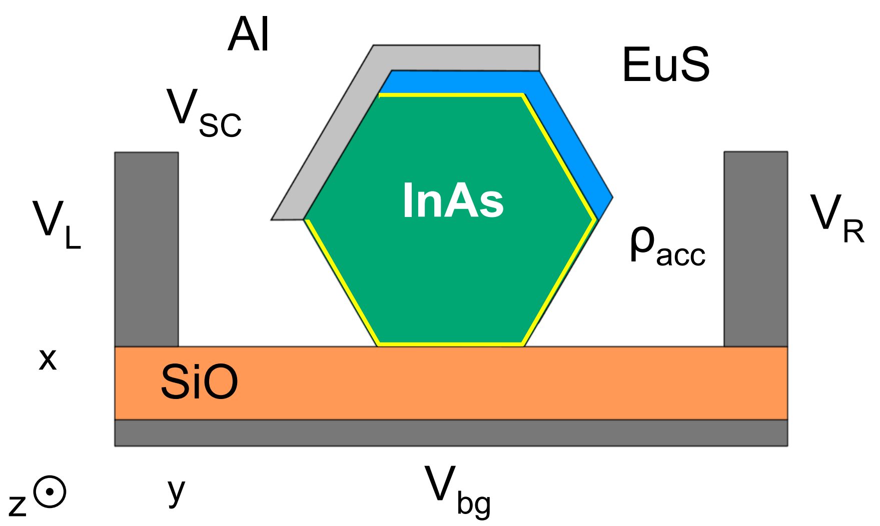
Overlapping device
(shows ZBP)

Non-overlapping device
(doesn't show ZBP)

Realistic model

We include in the Hamiltonian all the materials involved in the heterostructure using realistic parameters. We also include the self-consistent electrostatic environment.

$$H = \vec{k}\frac{\hbar^2}{2m_{\rm eff}(\vec{r})}\vec{k} + E_{\rm F}(\vec{r}) - e\phi(\vec{r}) + h_{\rm ex}(\vec{r})\sigma_x + \Delta(\vec{r})\tau_x\sigma_x + \frac{1}{2}\left[\vec{\alpha}(\vec{r})\cdot(\vec{\sigma}\times\vec{k}) + (\vec{\sigma}\times\vec{k})\cdot\vec{\alpha}(\vec{r})\right]$$



InAs	
m_{eff}	$0.023m_0$
E_F	0
h_{ex}	0
Δ	0
α_R	

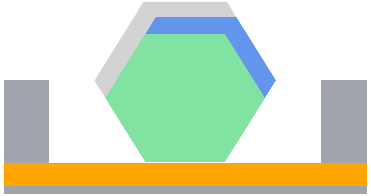
EuS	
m_{eff}	$0.3m_0$
E_F	0.7eV
h_{ex}	0.1eV
Δ	0
α_R	0

Al	
m_{eff}	m_0
E_F	-10eV
h_{ex}	0
Δ	0.23meV
α_R	0

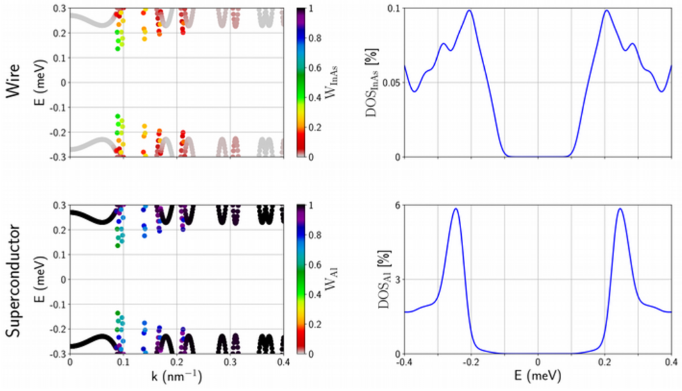
$$\vec{\alpha}(\vec{r}) = \vec{\alpha}_{\rm int} + \frac{eP_{\rm fit}^2}{3}\left[\frac{1}{\Delta_{\rm g}^2} - \frac{1}{(\Delta_{\rm g}+\Delta_{\rm soff})^2}\right]\vec{\nabla}\phi(\vec{r})$$

Realistic model

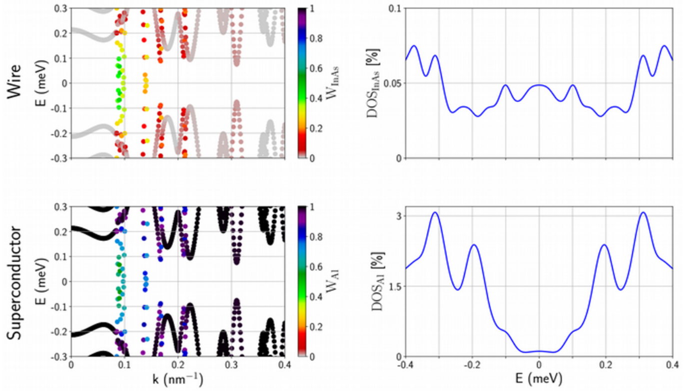
We compute the energy spectrum versus the momentum k_z for the **overlapping device** fixing all the gates to $V_i=0$. From there, we also compute the DOS. We perform three different simulations.



$\alpha_R=0 \quad h_{ex}=0$



$\alpha_R=0 \quad h_{ex}\neq0$



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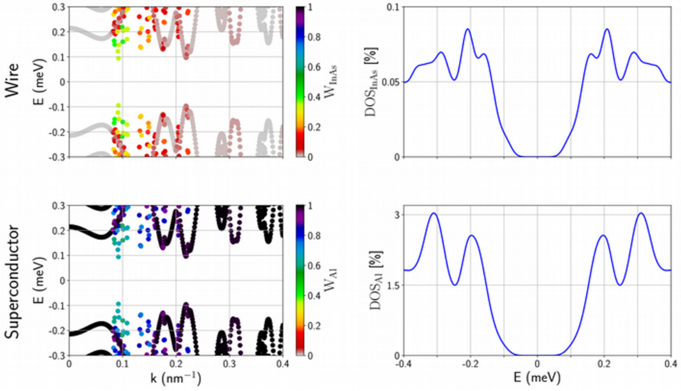
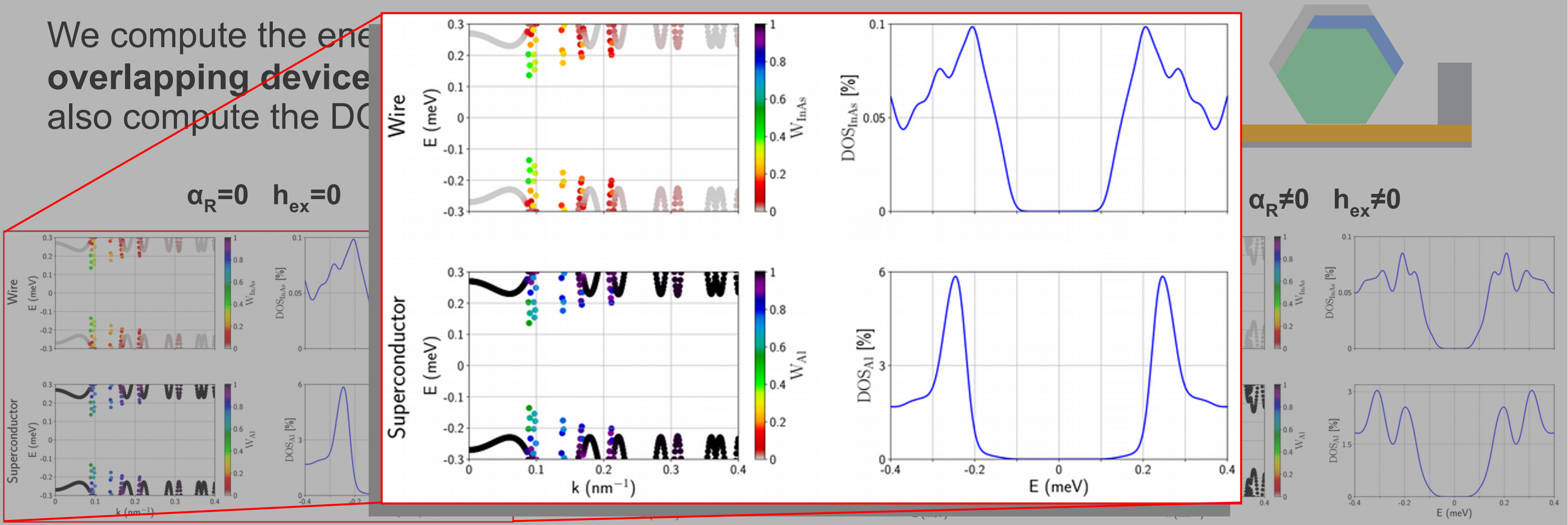


Realistic model

We compute the ene
overlapping device
also compute the DO

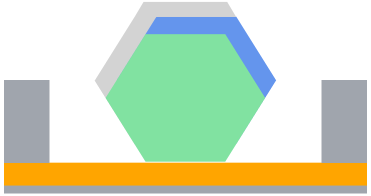
α_R=0 h_{ex}=0

α_R≠0 h_{ex}≠0



Realistic model

We compute the energy spectrum versus the momentum k_z for the **overlapping device** fixing all the gates to V_i=0. From there, we also compute the DOS. We perform three different simulations.



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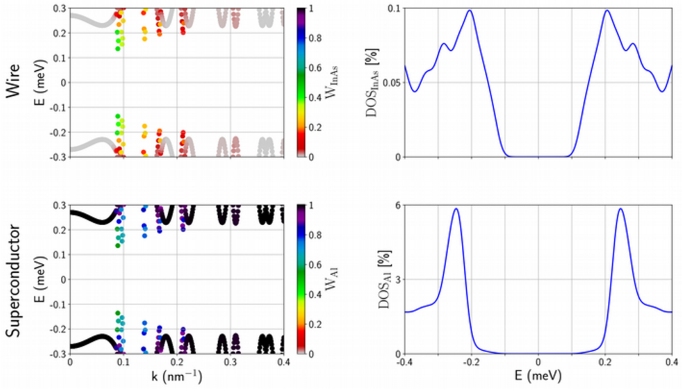
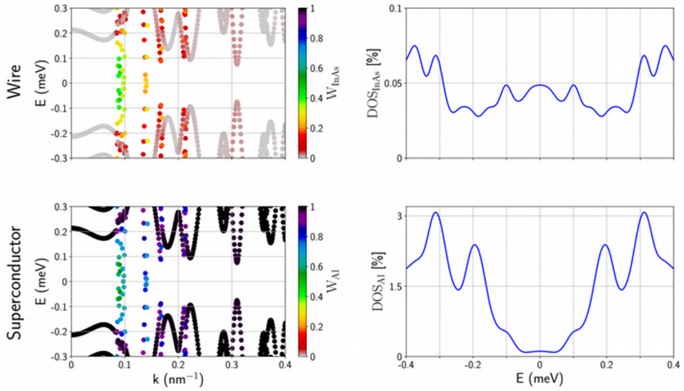


α_R=0 h_{ex}≠0



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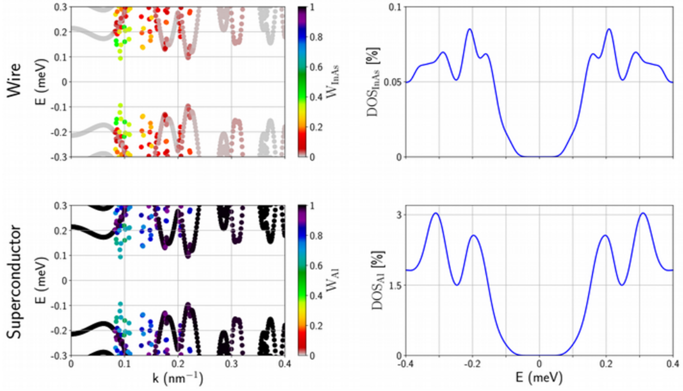

Induced gap
of 0.2 meV

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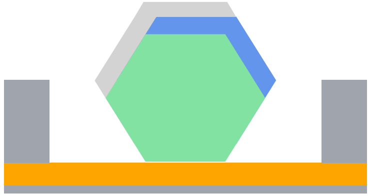

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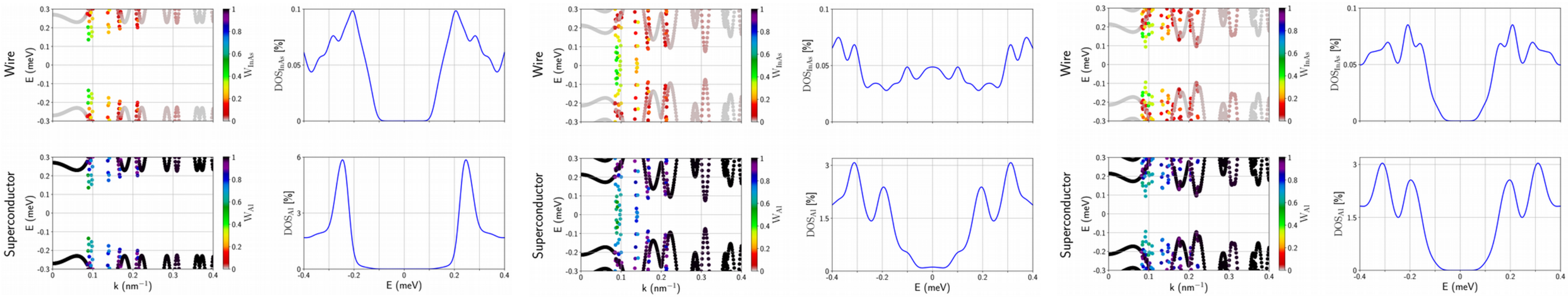

Without SOC, the gap is closed in the NW → The induced V_Z is larger than the induced Δ

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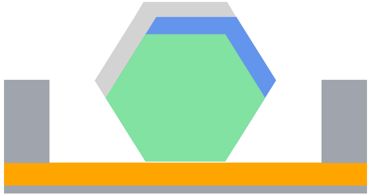

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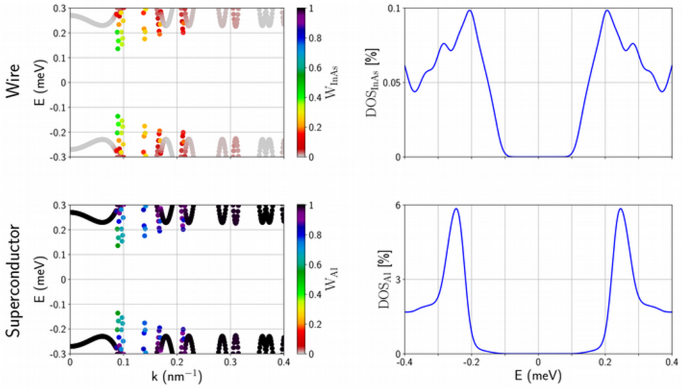

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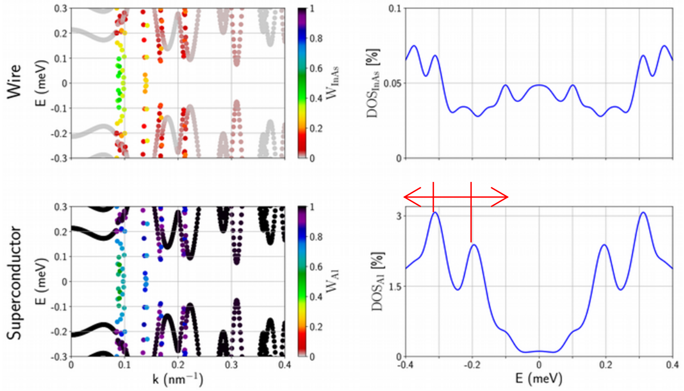

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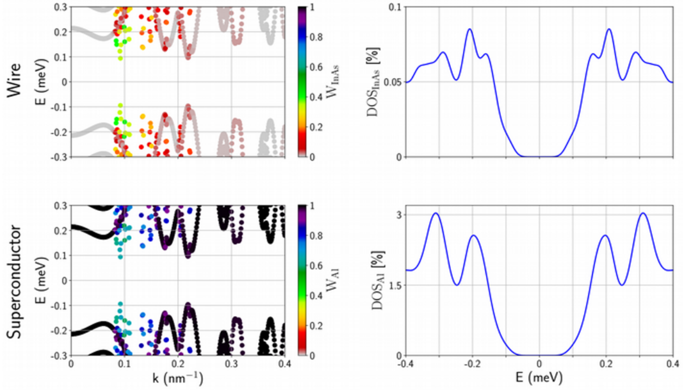

A small exchange field of 0.06meV is also induced in the SC, as previous experiments showed

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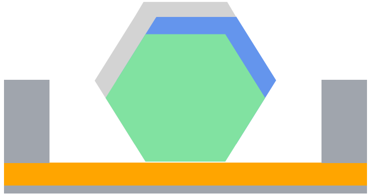

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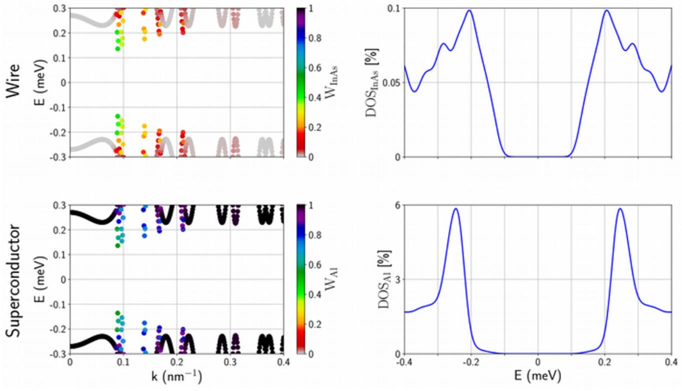

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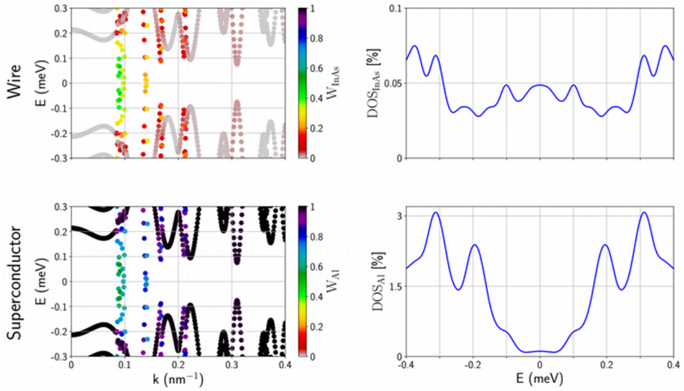

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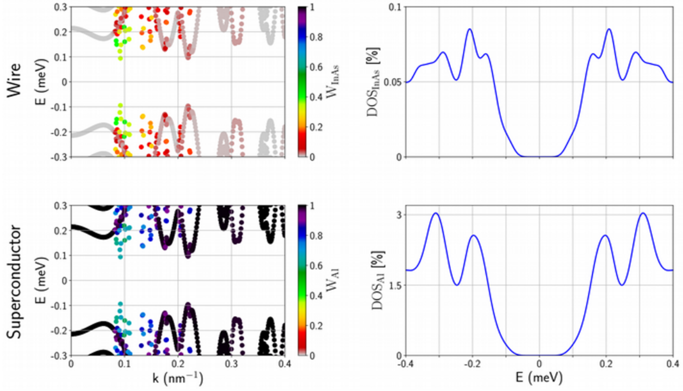
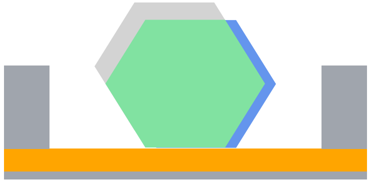
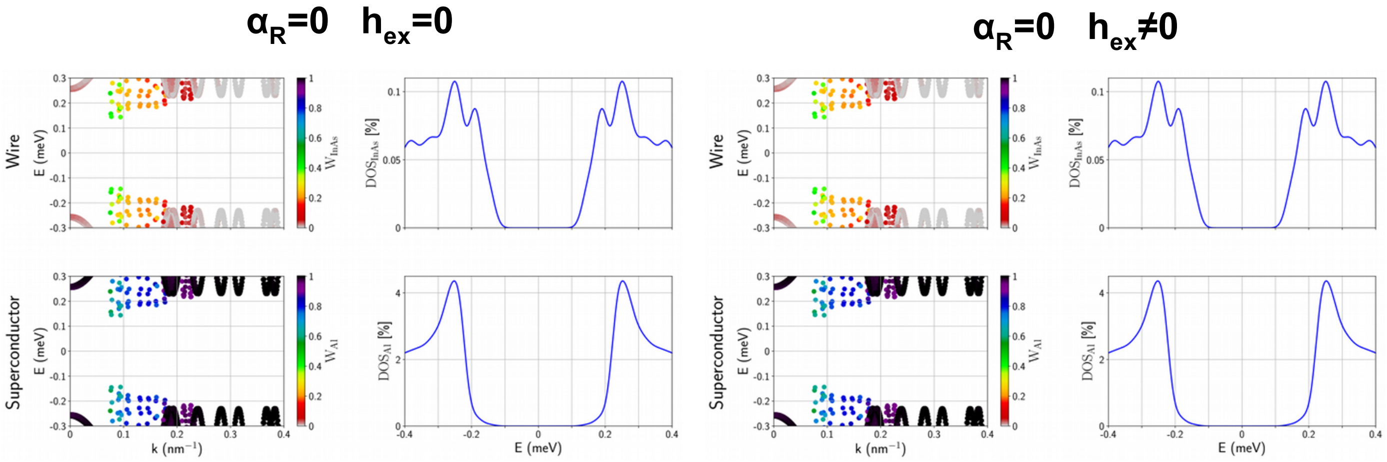


Signature of topological phase transition ← With SOC, the gap reopens

Realistic model

We perform exactly the same simulations but for the **non-overlapping device**.





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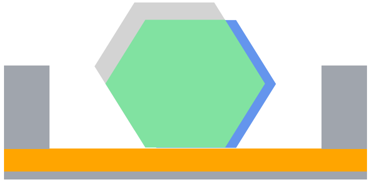

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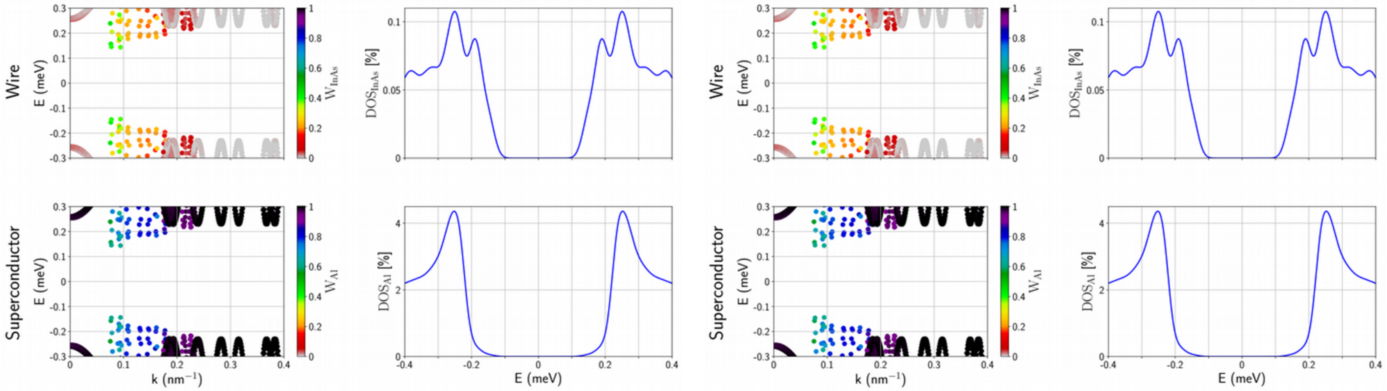

For the non-overlapping device, the induced exchange field seems not to be large enough to close the gap

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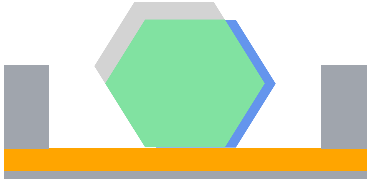

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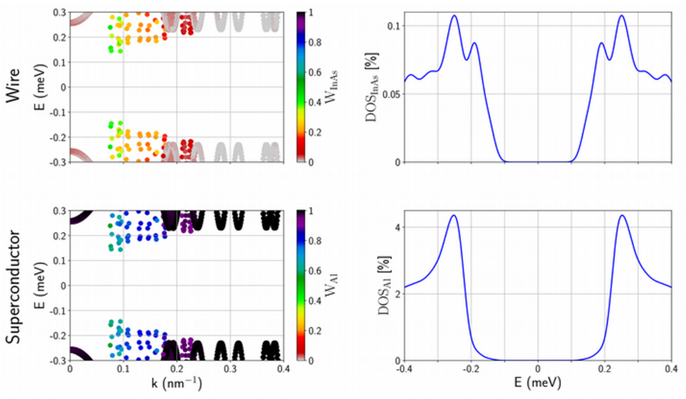

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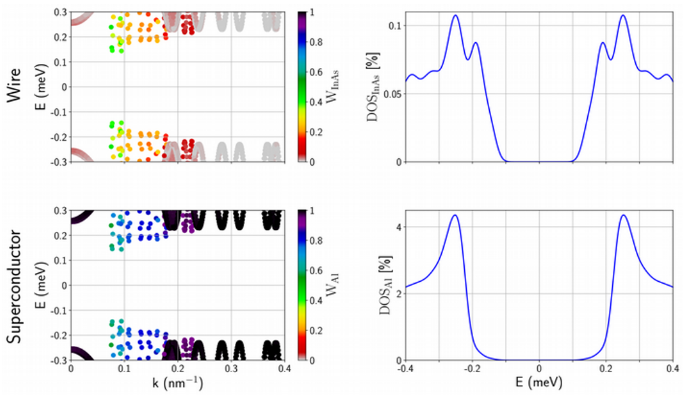

There is no topological phase in the non-overlapping device, at least for this gate voltage

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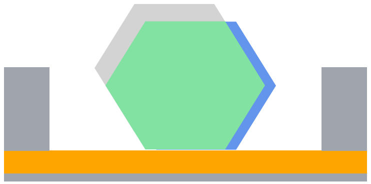

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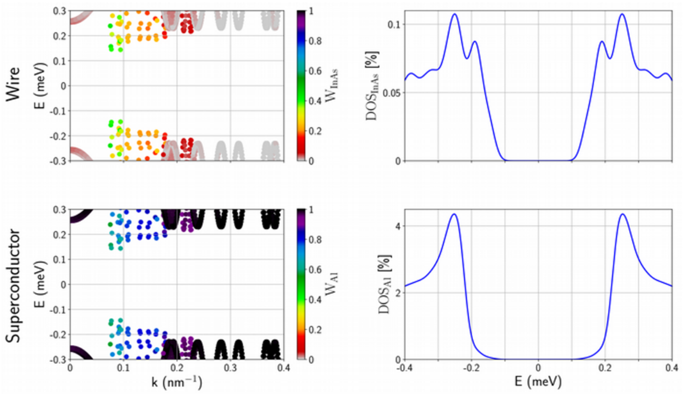

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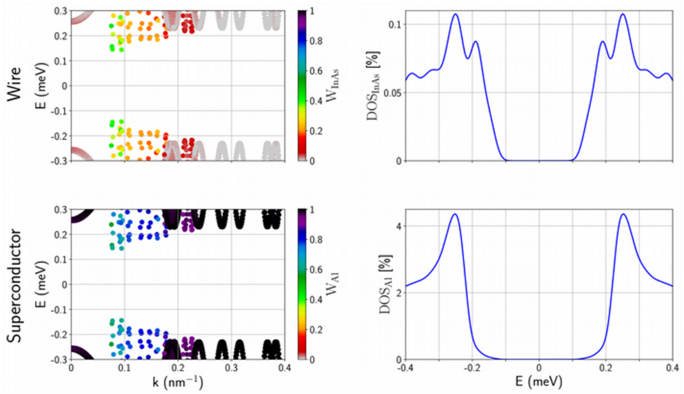


Topological phase diagram?

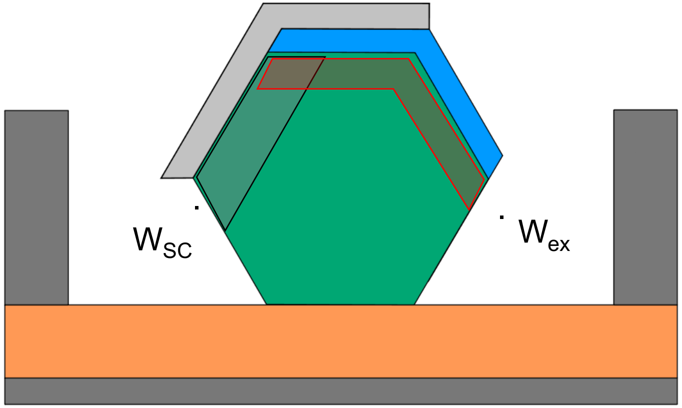
There is no topological phase in the non-overlapping device, at least for this gate voltage

For the non-overlapping device, the induced exchange field seems not to be large enough to close the gap

Effective model

We “integrate out” the Al and the EuS, and we directly include the proximity effects into the InAs nanowire in an effective way. This reduces the computational cost and allows to find the phase diagram.

There is only a superconducting pairing of Δ=0.2meV in this proximitizing region (W_{SC}=30nm), as well as an exchange field of h_{ex}=0.06meV



There is an exchange field of $h_{ex}^{(EuS)}$=0.1eV in this proximitizing region (W_{ex}=1nm)

$$H = \frac{\hbar^2 k^2}{2m_{\rm eff}} + E_{\rm F} - e\phi(\vec{r}) + h_{\rm ex}(\vec{r})\sigma_x + \Delta(\vec{r})\tau_x\sigma_x + \frac{1}{2}\left[\vec{\alpha}(\vec{r})\cdot(\vec{\sigma}\times\vec{k}) + (\vec{\sigma}\times\vec{k})\cdot\vec{\alpha}(\vec{r})\right]$$

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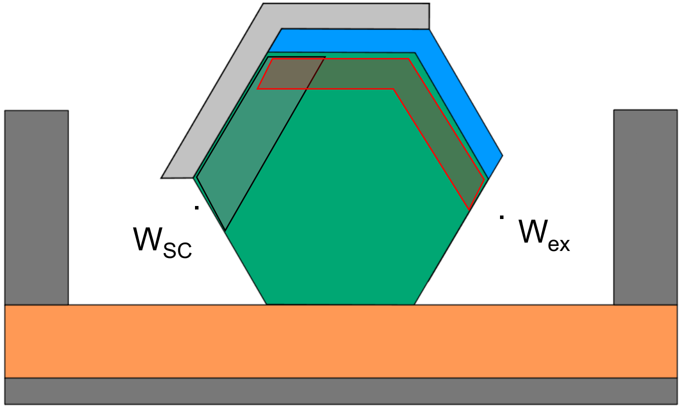
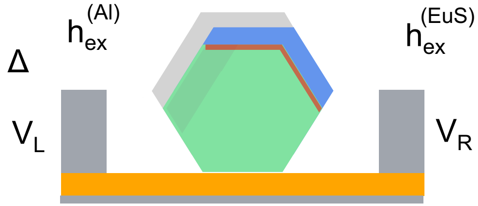


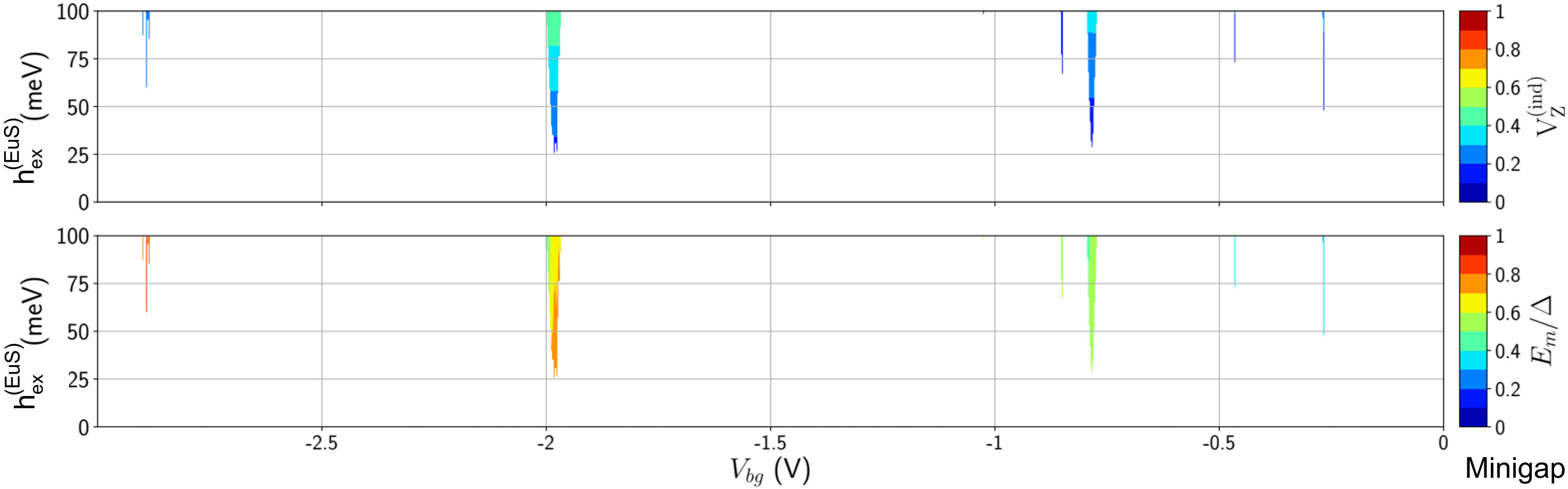
There is an exchange field of $h_{ex}^{(EuS)}$=0.1eV in this proximitizing region (W_{ex}=1nm)

We compute the induced magnetization and superconductivity. We choose W_{SC} and W_{ex} in such a way to reproduce (roughly) the same behaviour as in the realistic model.

Effective model

Phase diagram vs $\mathbf{V_{bg}}$ (fixing V_L=0 and V_R=-4V) for an **overlapping** device with direct-induced magnetization





Minigap

Effective model

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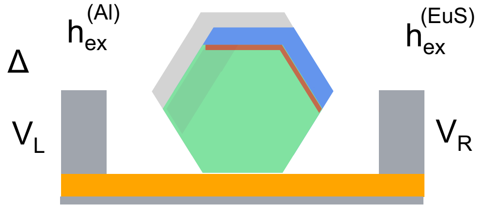

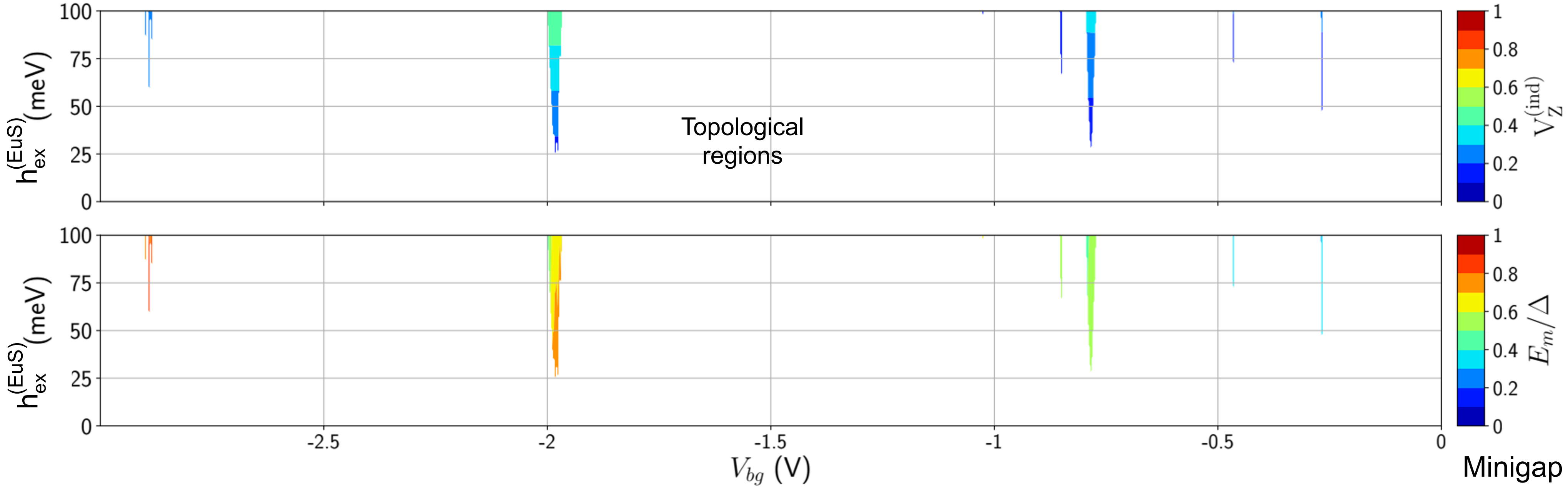

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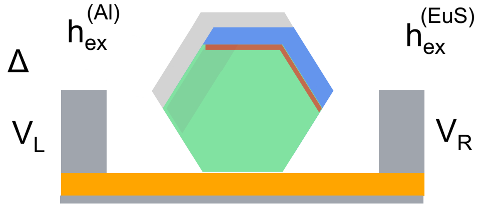
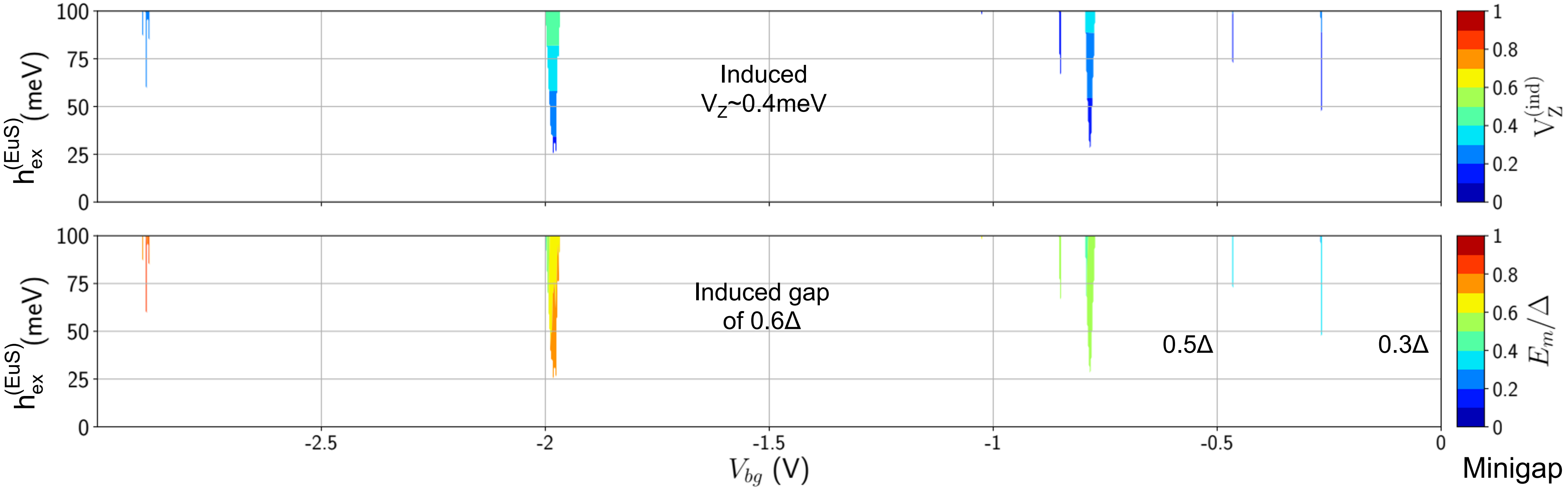
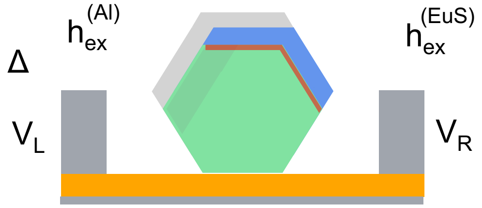
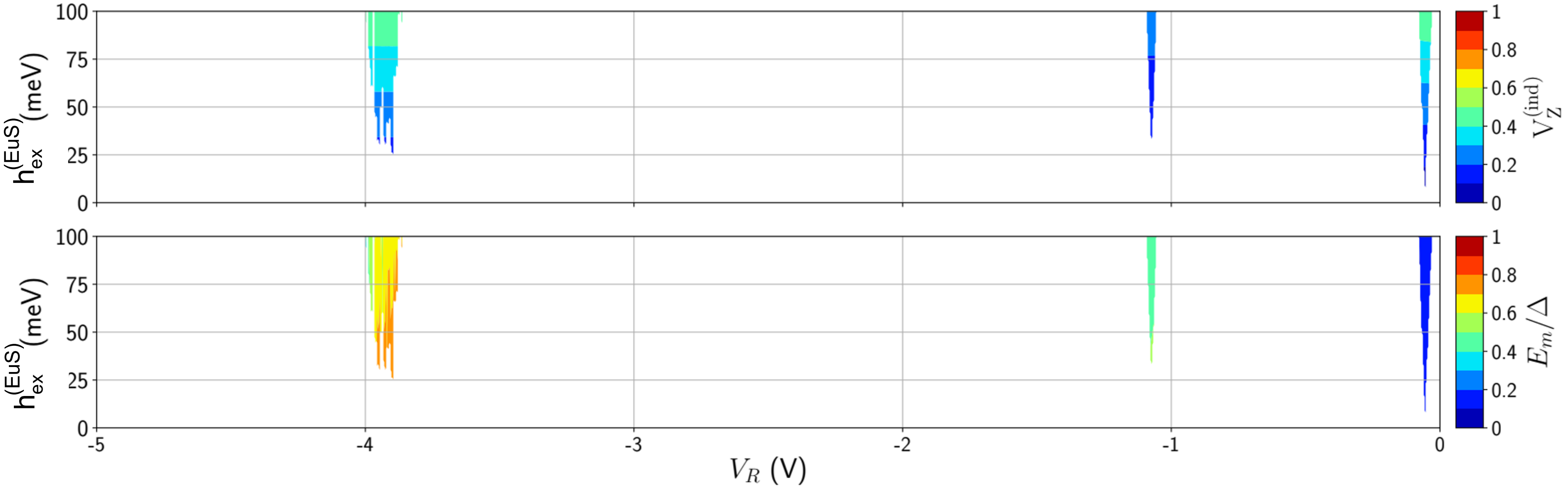




Effective model

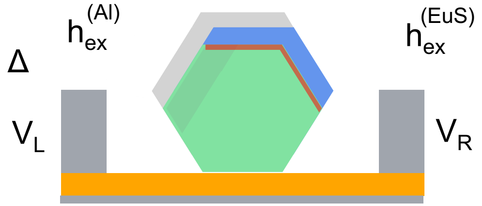
Phase diagram vs $\mathbf{V_R}$ (fixing V_L=0 and V_{bg}=-2V) for an **overlapping** device with direct-induced magnetization

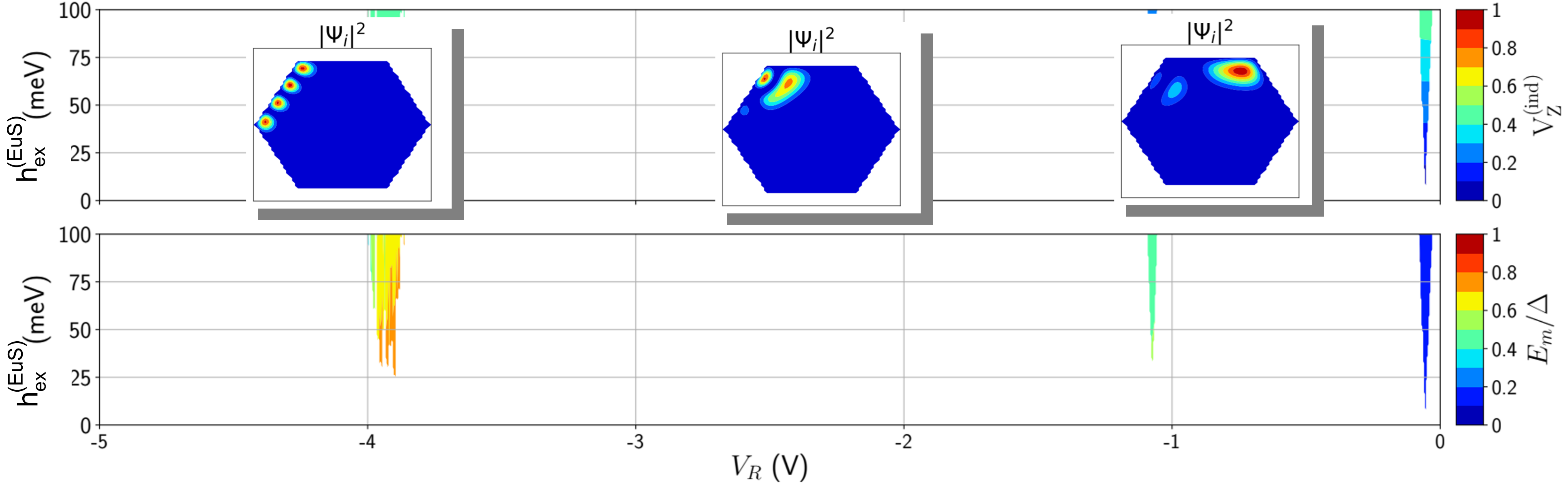




Effective model

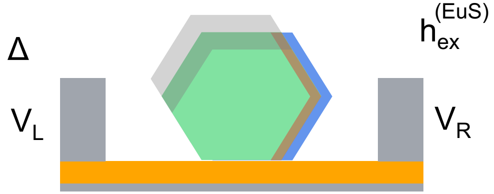
Phase diagram vs $\mathbf{V_R}$ (fixing V_L=0 and V_{bg}=-2V) for an **overlapping** device with direct-induced magnetization

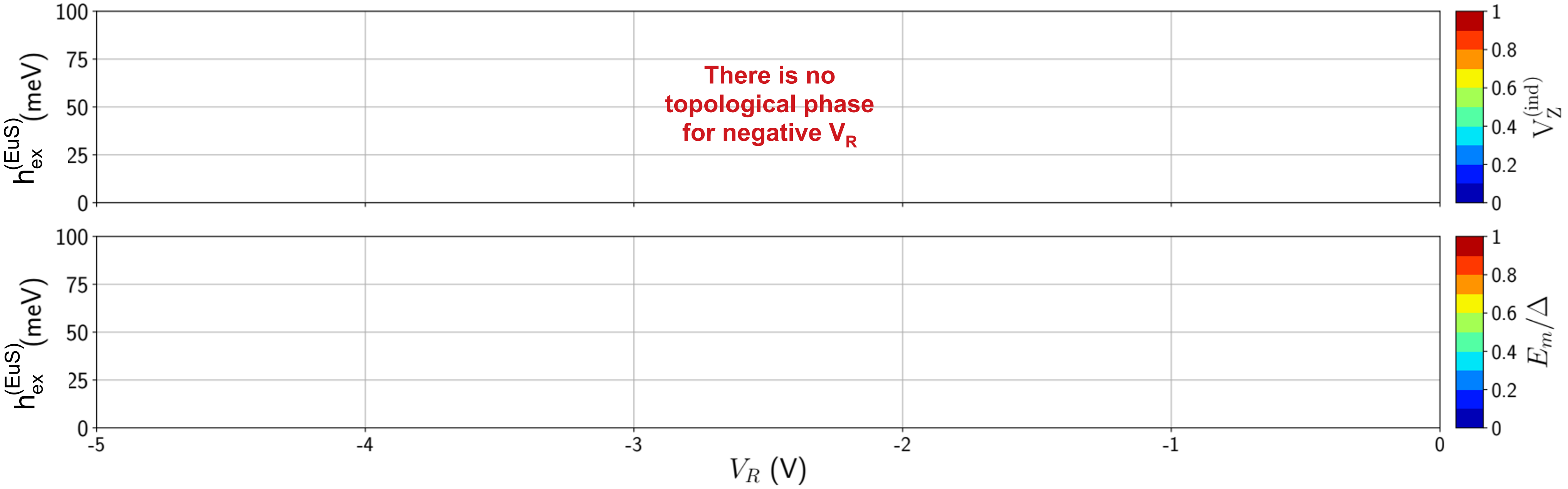




Effective model

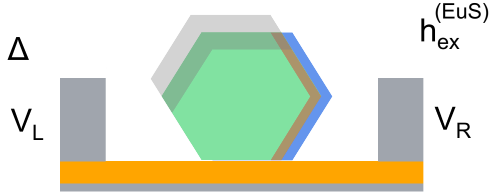
Phase diagram vs $\mathbf{V_R}$ (fixing V_L=0 and V_{bg}=-2V) for a **non-overlapping** device with direct-induced magnetization

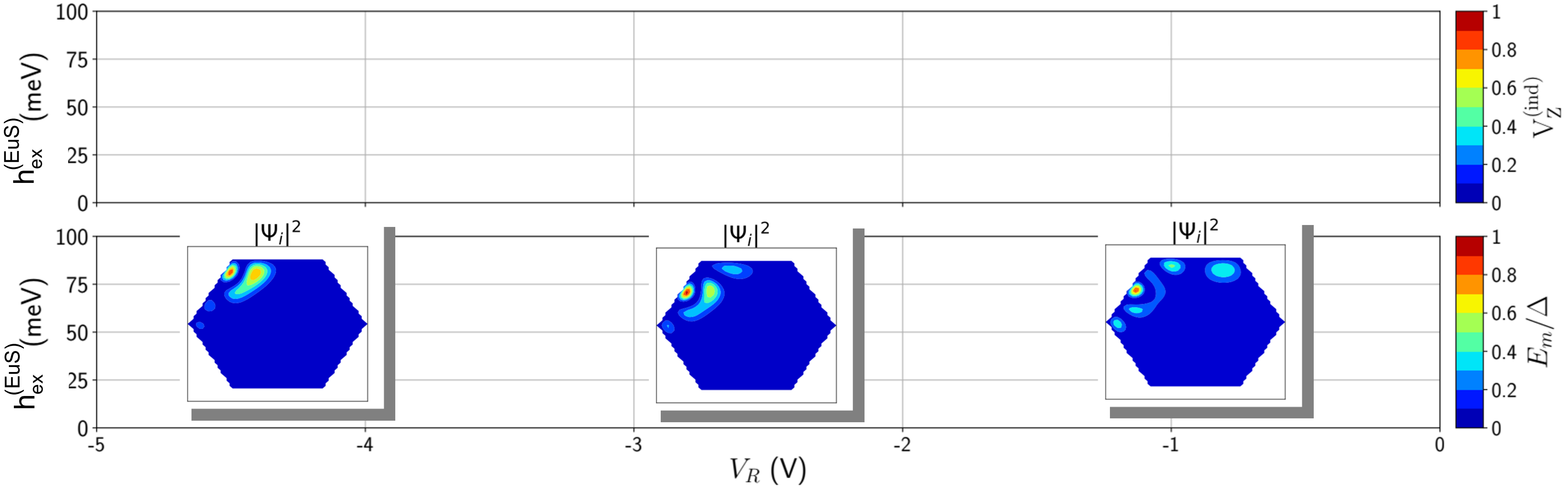




Effective model

Phase diagram vs $\mathbf{V_R}$ (fixing V_L=0 and V_{bg}=-2V) for a **non-overlapping** device with direct-induced magnetization





Conclusions and outlook

Conclusions

- InAs/Al/EuS heterostructures intrinsically incorporates the effect of a Zeeman field large enough so that they can support MBS.

- Only some specific geometries give rise to MBS, because the wavefuntion needs to be close to the EuS-InAs and Al-InAs interfaces at the same time. The strength of the proximity effects can be controlled by the gates.

Reference

- Microscopic analysis of topological superconductivity in ferromagnetic hybrid nanowires, Samuel D. Escribano, Elsa Prada, Yuval Oreg and Alfredo Levy Yeyati, arXiv:2011.06566 (2020).

For any question or inquire, don't hesitate to contact me via email at **samuel.diaz@uam.es**, thank you for your attention!

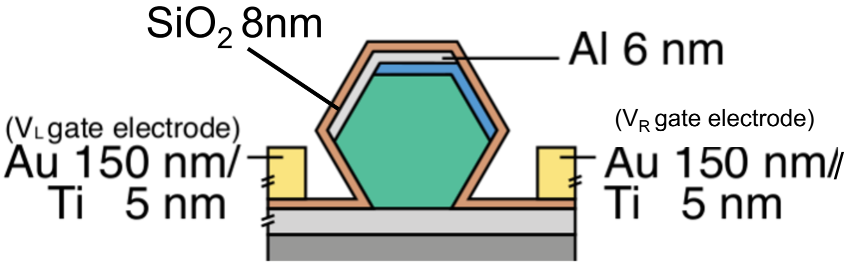
Supplementary Material

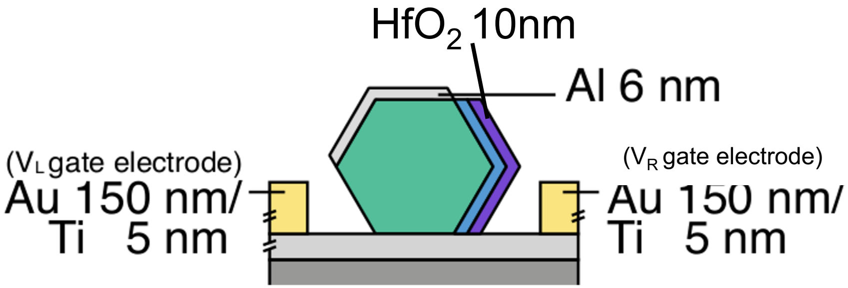
A: Effective Model

Model

The electrostatic potential is determined self-consistently (in the Thomas-Fermi approximation) using the Poisson equation. The electrostatic environment is taken into account through the dielectric permittivity.

$$\vec{\nabla}(\epsilon(\vec{r}) \cdot \vec{\nabla}\phi(\vec{r})) = \rho(\vec{r})$$



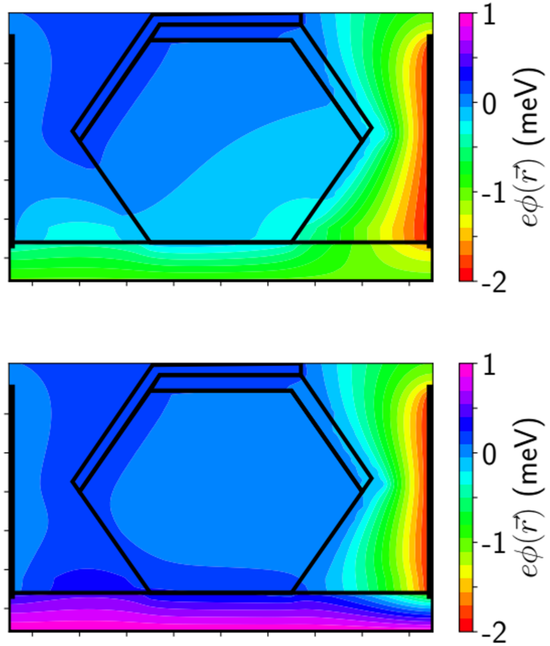


A recent experiment shows that there is an accumulation layer at the InAs-EuS interface similar to the one of the free facets. Thus, we include the same accumulation layer ρ_{acc} in the nanowire facets that are not in contact with Al. Additionally, we simulate the InAs-Al band bending imposing V_{SC} as boundary condition on the Al.

Model

Electrostatic potential
Induced superconductivity
Induced Zeeman field

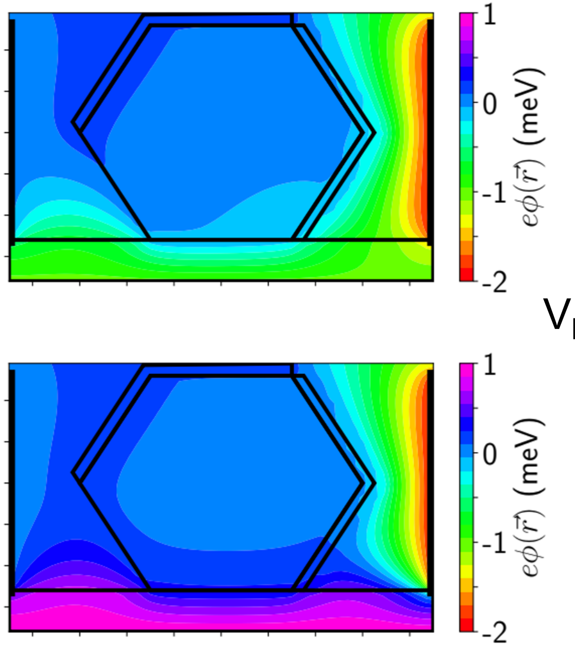
Overlapping device



V_{bg}=-1V

V_{bg}=1V

Non-overlapping device



V_{bg}=-1V

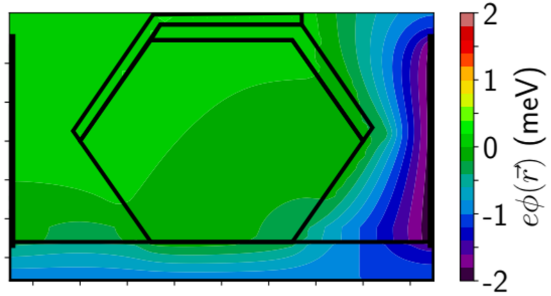
V_{bg}=1V

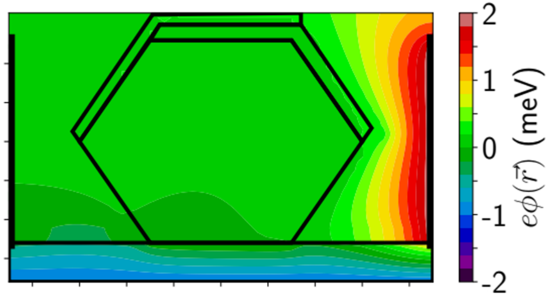
As the back-gate voltage is increased, the wavefunction is pushed towards the bottom of the wire.

Model

Overlapping device

V_R=-2V



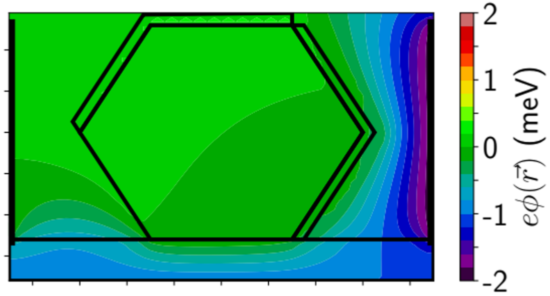


V_R=2V

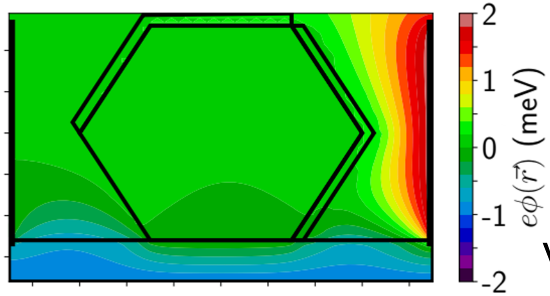
As the right-gate voltage is increased the wavefunction is pushed towards the EuS.

The proximity effects, both with Al and EuS can thus be controlled by the gates.

Non-overlapping device



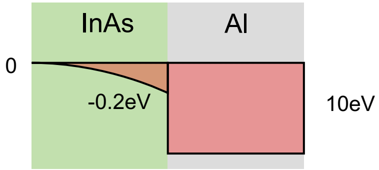
V_R=-2V



V_R=2V

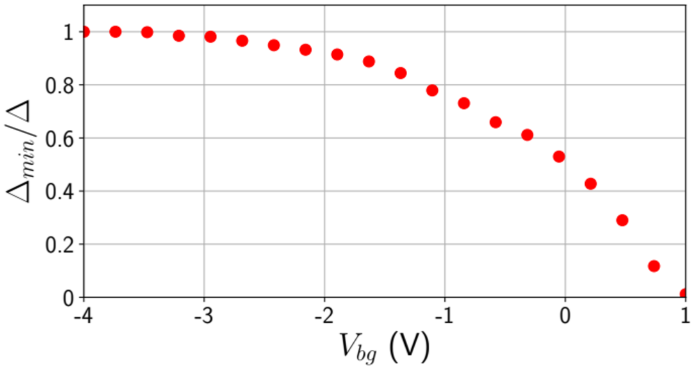
Model

To describe the superconductivity inside the semiconductor, one would need, in principle, to include the superconducting layer also at a tight-binding level.



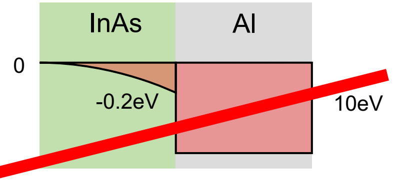
The SC is described as a metallic region (with a band-offset of -10eV) with a paring amplitude Δ

One can obtain the spectra of the system for different gates, and from there, the DOS in the wire and the induced gap (the minimum gap Δ_{min}).

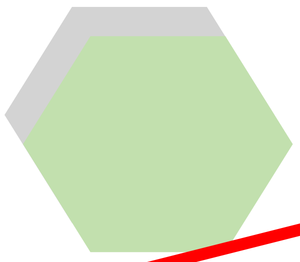


Model

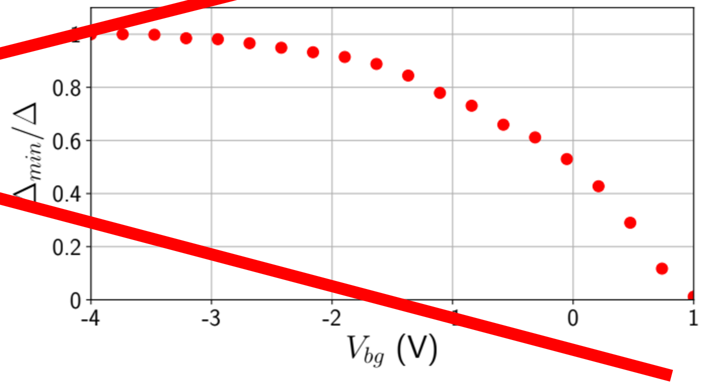
To describe the superconductivity inside the semiconductor, one would need, in principle, to include the superconducting layer also at a tight-binding level.



The SC is described as a metallic region (with a band-offset of -10eV) with a paring amplitude Δ



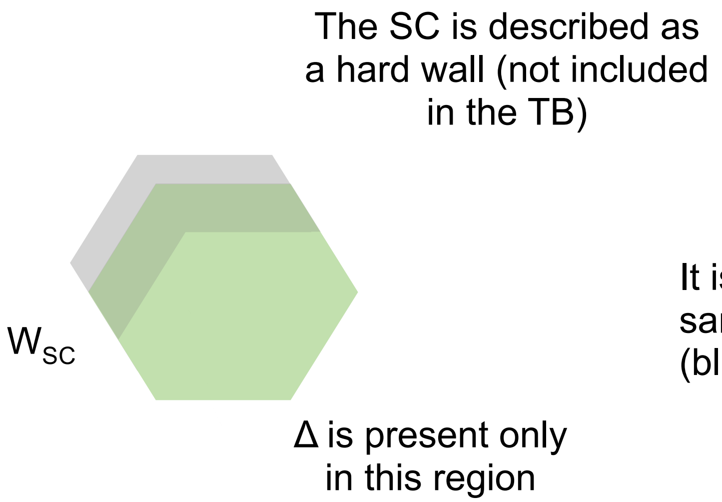
One can obtain the spectra of the system for different gates, and from there, the DOS in the wire and the induced gap (the minimum gap Δ_{min}).



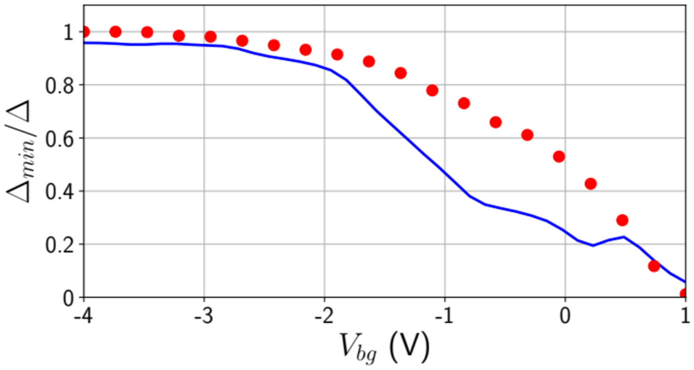
Unfortunately, this is not computationally affordable.

Model

A different approach to include the proximity effect in the wire is to assume that a region of width W_{SC} close to the InAs/Al interface is characterized by a paring amplitude Δ.



It is possible to do the same for this system (blue line).

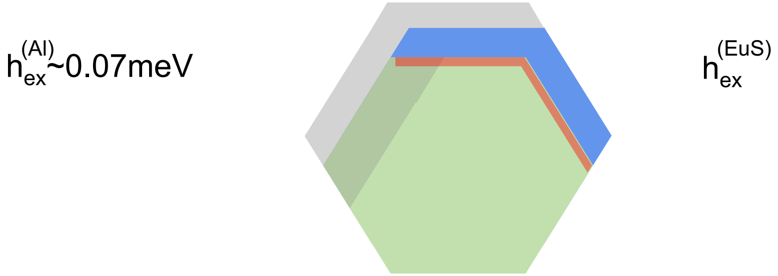


Using W_{SC}=30nm we predict a similar behaviour.

Model

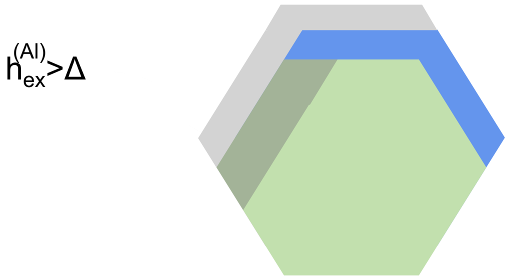
It is not clear how the magnetization induced by the EuS influences the state of the nanowire. There are two possible scenarios, which could be complementary.

Model 1: direct-induced magnetization



The EuS **directly** induces an exchange field ($h_{ex}^{(EuS)}$) in the InAs. Because the EuS is an insulator, the proximitized region is small (1nm), but with a large exchange field. In addition, it is known that there is a small exchange field ($h_{ex}^{(Al)}$=0.07meV) in the Al due to the Al/EuS interface.

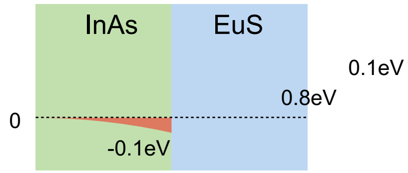
Model 2: indirect-induced magnetization



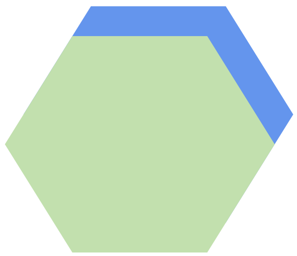
The EuS induces an exchange field $h_{ex}^{(Al)}$ in the InAs through the Al layer in an **indirect** way. The exchange field induced in the SC due to the Al-EuS interface is indeed, for whichever reason, larger than Δ. The spin-orbit coupling opens a gap even if the Clogston limit is reached.

Model

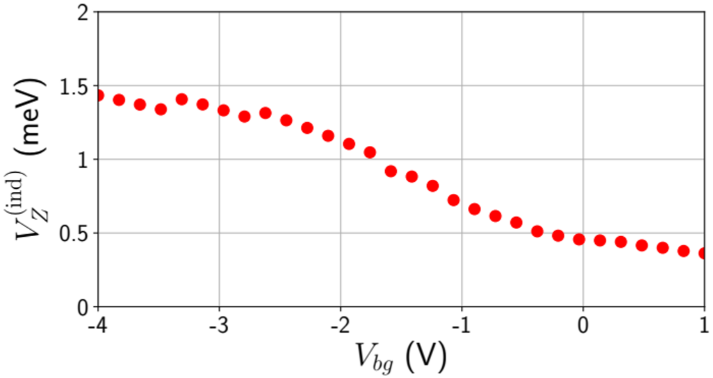
To show that the first model is also plausible, let us describe first the EuS at a tight-binding level as well.



The EuS is described as an insulating region (with a band-offset of 0.8eV) characterized by a large exchange field h_{ex} (with a Zeeman splitting of 0.1eV)



One can obtain the spectra of the system for different gates, and from there, the induced magnetization in the wire.



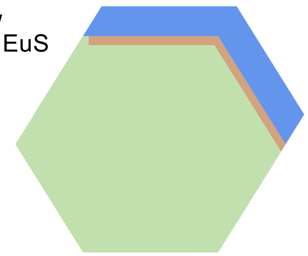
Model

Electrostatic potential
Induced superconductivity
Induced Zeeman field

Although it is (computationally) affordable to include the EuS at a tight-binding layer, let us describe it as a proximitized region close to the InAs-EuS interface, as we did for the Al.

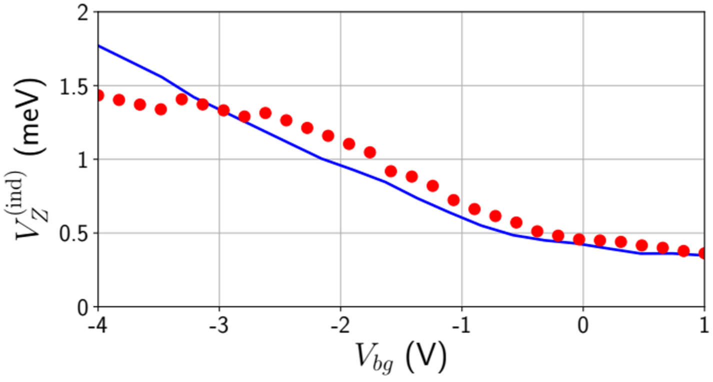
The EuS is described as a hard wall (not included in the TB)

W_{EuS}



h_{ex} is present only in this region

It is possible to do the same for this system (blue line).



Using W_{EuS}=1nm and $h_{ex} \simeq$ 100meV we predict a similar behaviour.

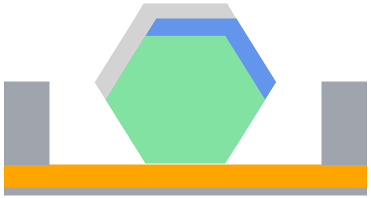
Supplementary Material

B: DOS vs V_{bg}

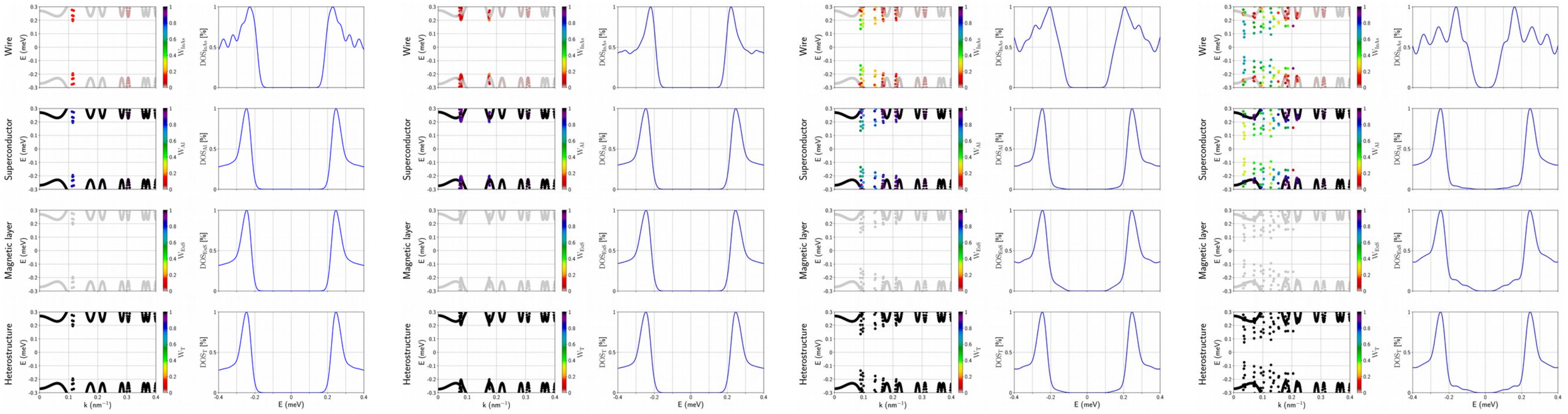
DOS vs V_{bg}

Overlapping device
Non-overlapping device

DOS vs V_{bg} for the **overlapping device** with $\mathbf{h_{ex}=0}$ and $\boldsymbol{\alpha_R=0}$.



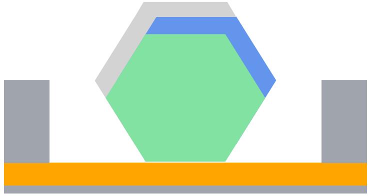
V_{bg}=-3V V_{bg}≈-1.5V V_{bg}≈0V V_{bg}=1V



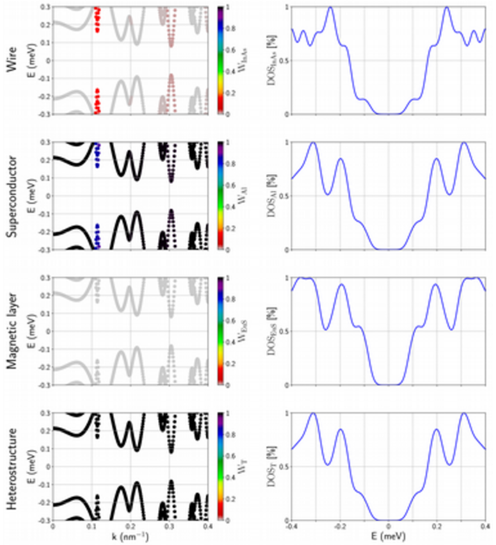
DOS vs V_{bg}

Overlapping device
Non-overlapping device

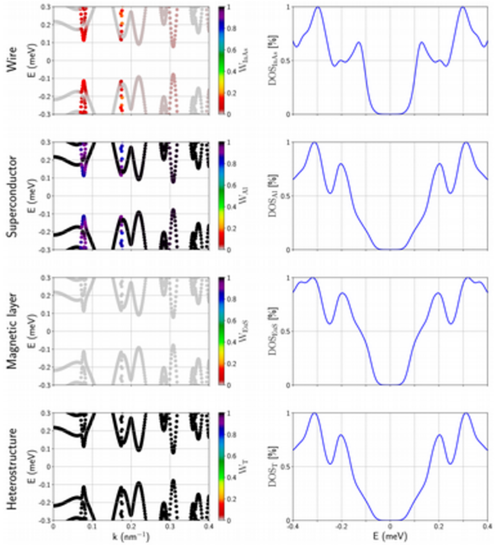
DOS vs V_{bg} for the **overlapping device** with $\mathbf{h_{ex} \neq 0}$ and $\boldsymbol{\alpha_R = 0}$.



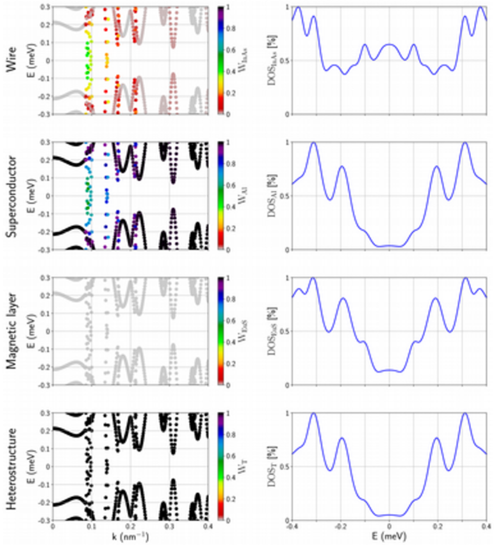
V_{bg}=-3V



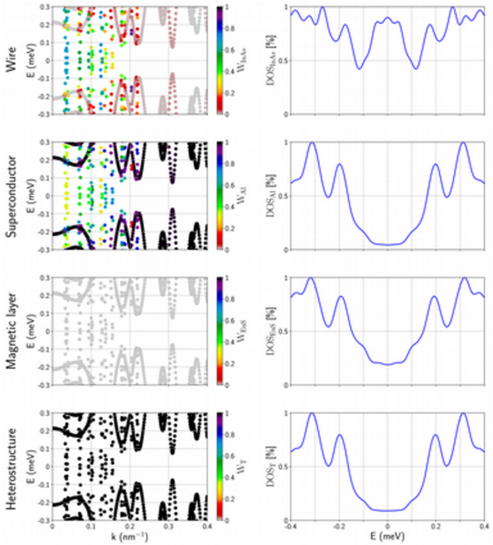
V_{bg}≈-1.5V



V_{bg}≈0V



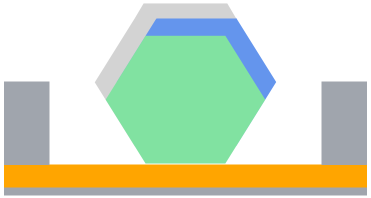
V_{bg}=1V



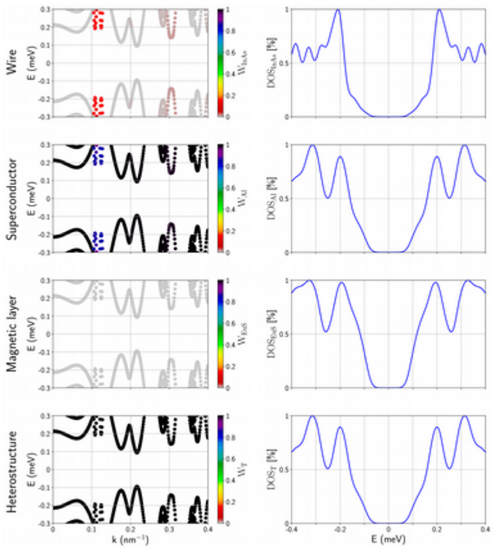
DOS vs V_{bg}

Overlapping device
Non-overlapping device

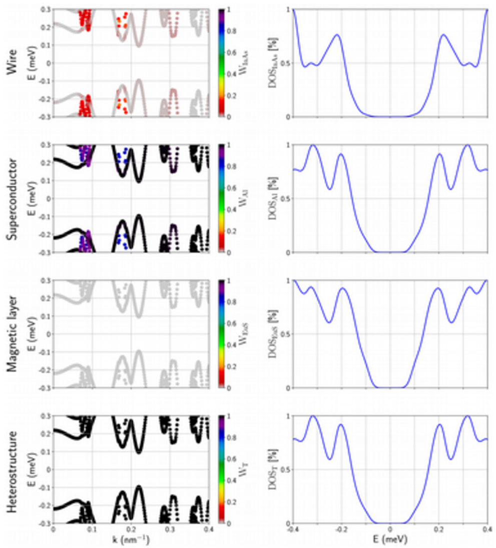
DOS vs V_{bg} for the **overlapping device** with $\mathbf{h_{ex}\neq0}$ and $\boldsymbol{\alpha_R\neq0}$.



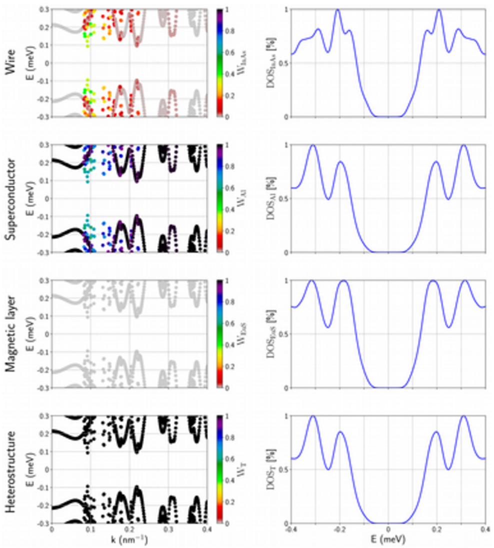
V_{bg}=-3V



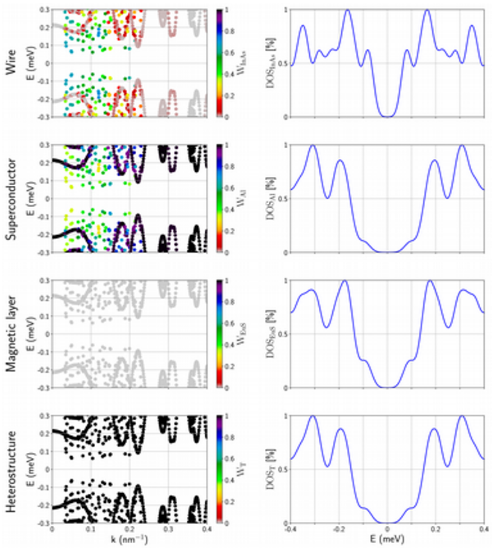
V_{bg}≈-1.5V



V_{bg}≈0V



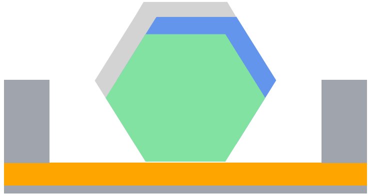
V_{bg}=1V



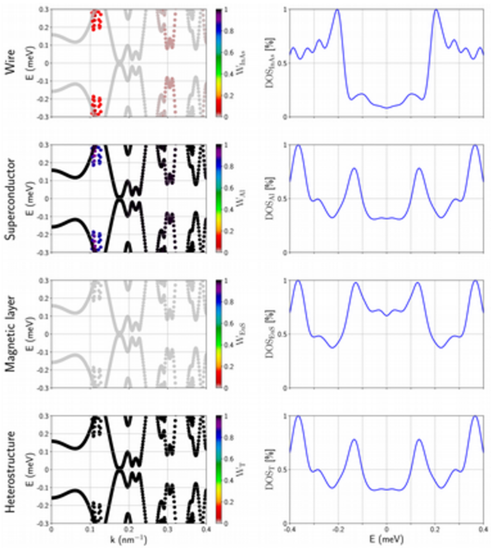
DOS vs V_{bg}

Overlapping device
Non-overlapping device

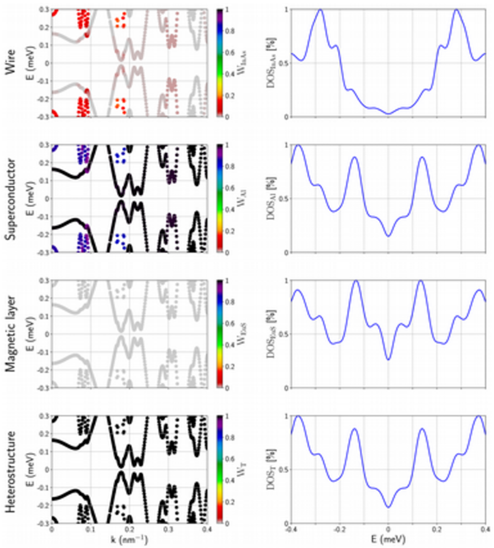
DOS vs V_{bg} for the **overlapping device** with **h_{ex}≠0 (double)** and **α_R≠0**.



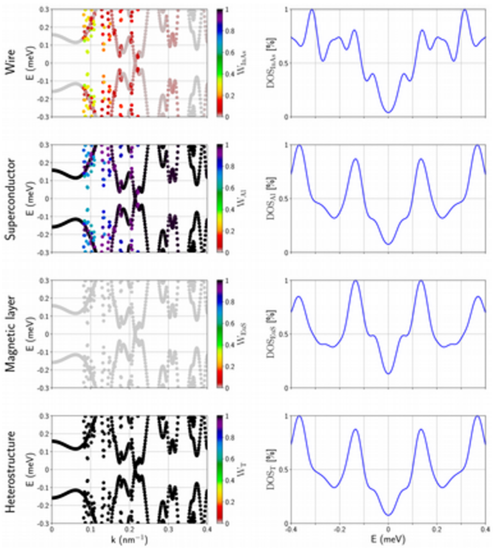
V_{bg}=-3V



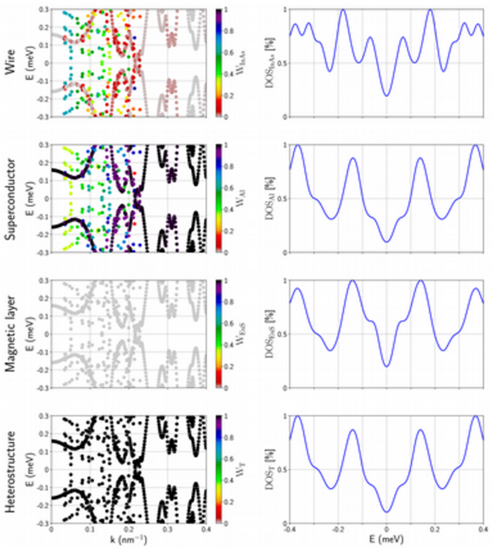
V_{bg}≈-1.5V



V_{bg}≈0V



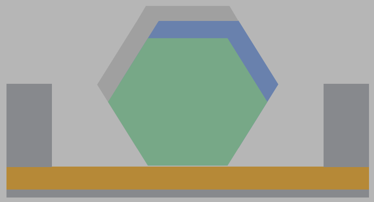
V_{bg}=1V

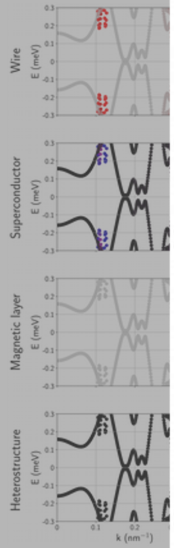


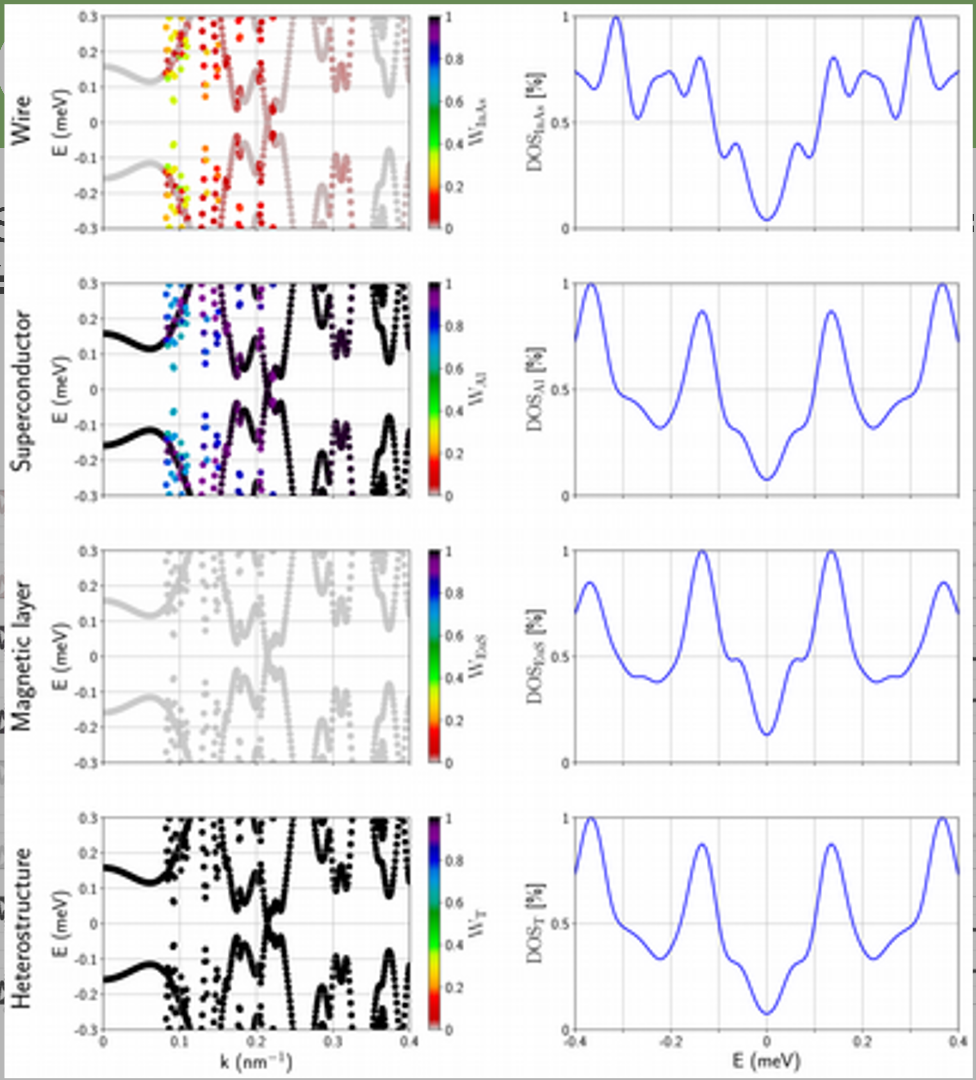
DO

Overlapping device
Non-overlapping device

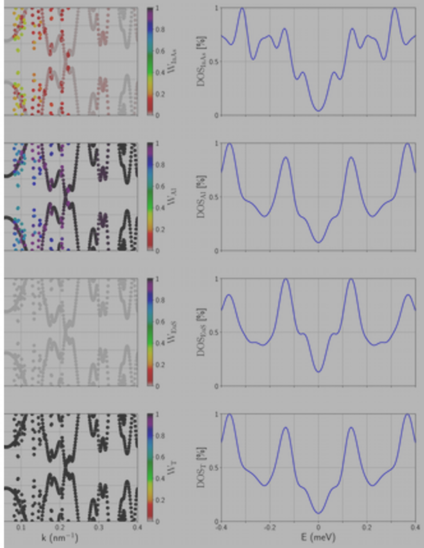
DOS … **0 (double)** and
$\alpha_R \neq$



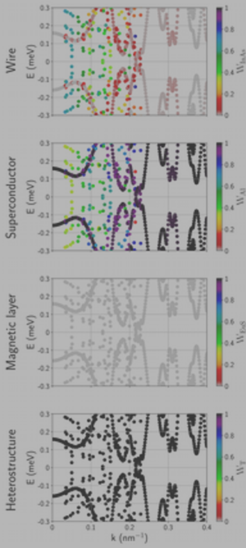


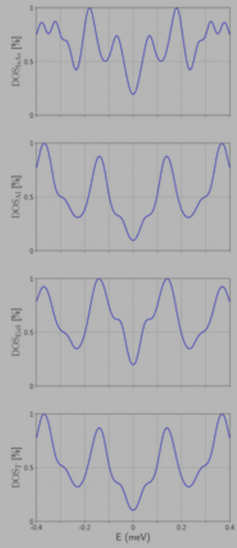


$V_{bg} \approx 0V$



$V_{bg} = 1V$

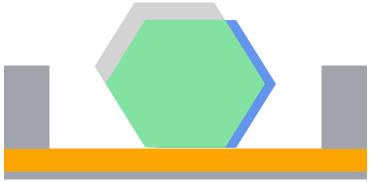




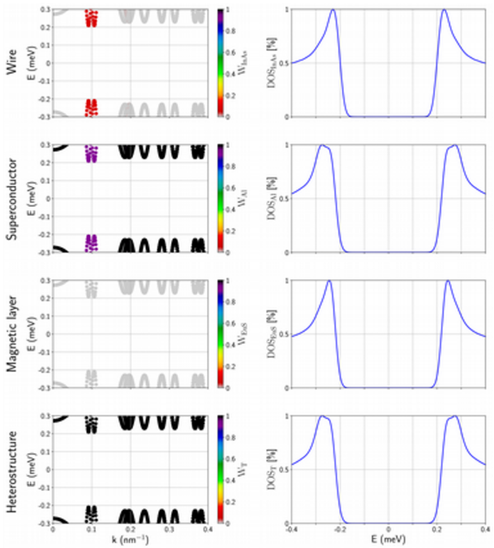
DOS vs V_{bg}

Overlapping device
Non-overlapping device

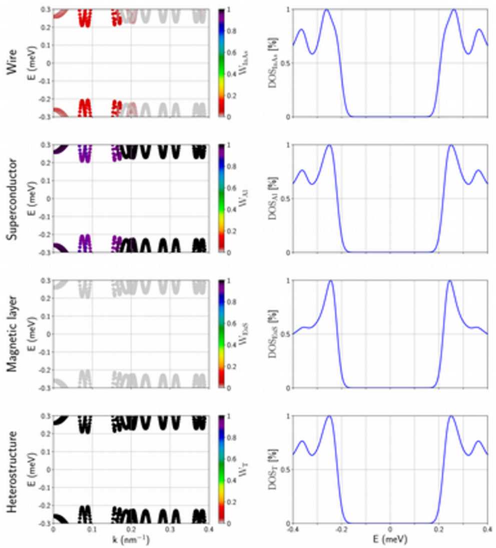
DOS vs V_{bg} for the **non-overlapping device**.



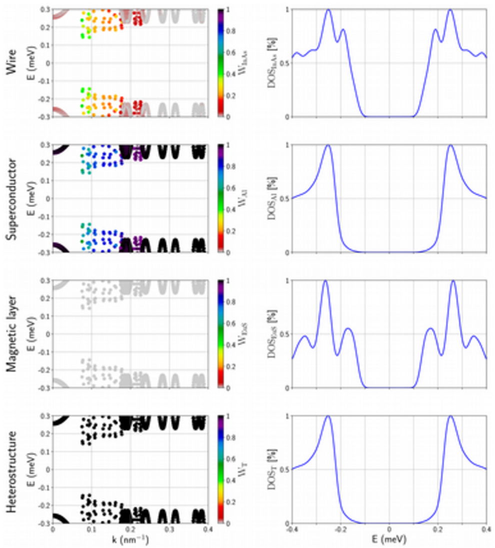
V_{bg}=-3V



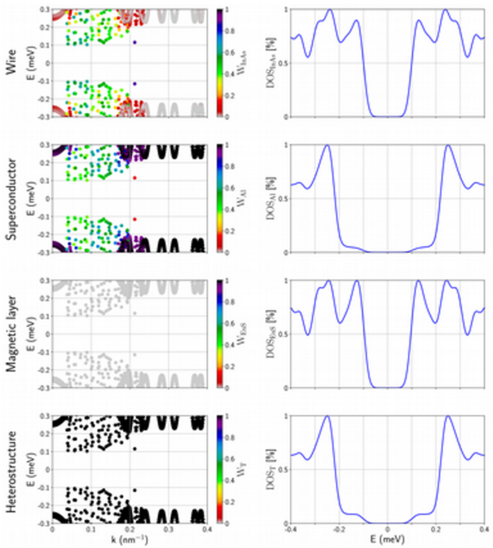
V_{bg}≈-1.5V



V_{bg}≈0V



V_{bg}=1V



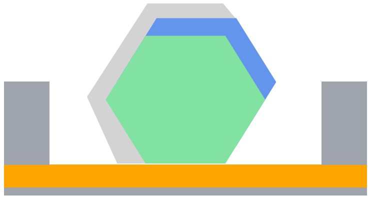
Supplementary Material

C: 4-facets geometry

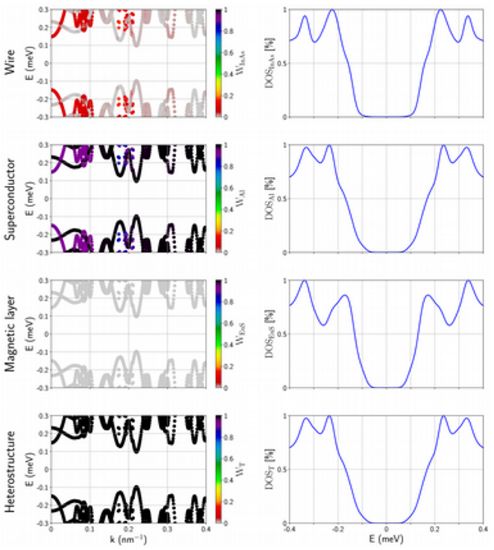
DOS vs V_{bg}

Overlapping device
Non-overlapping device

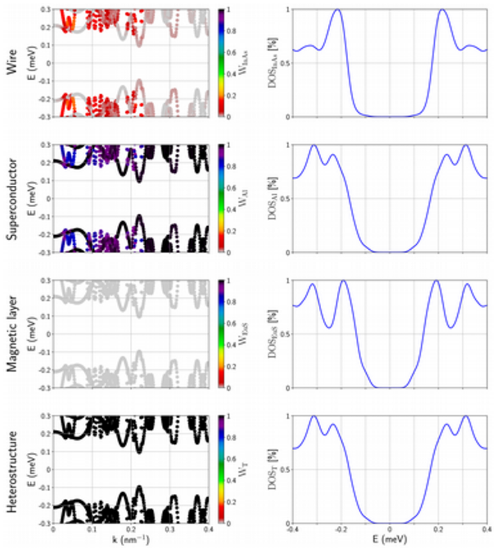
DOS vs V_{bg} for the **4-facets device** with **h_{ex}≠0** and **α_R=0**.



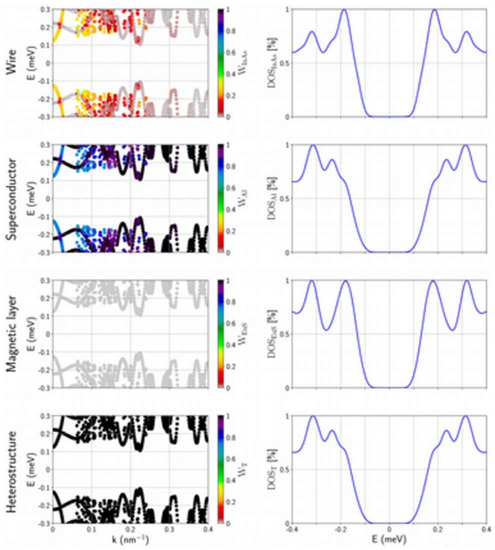
V_{bg}=-3V



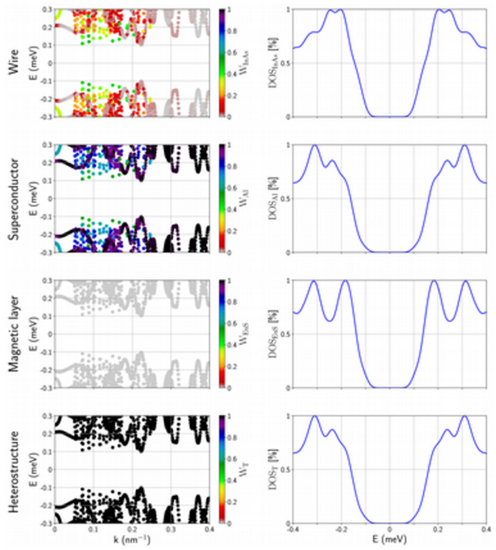
V_{bg}≈-1.5V



V_{bg}≈0V



V_{bg}=1V

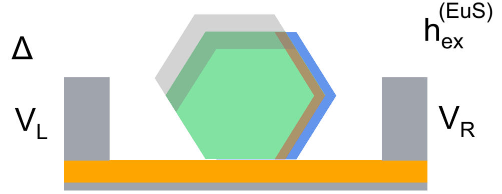


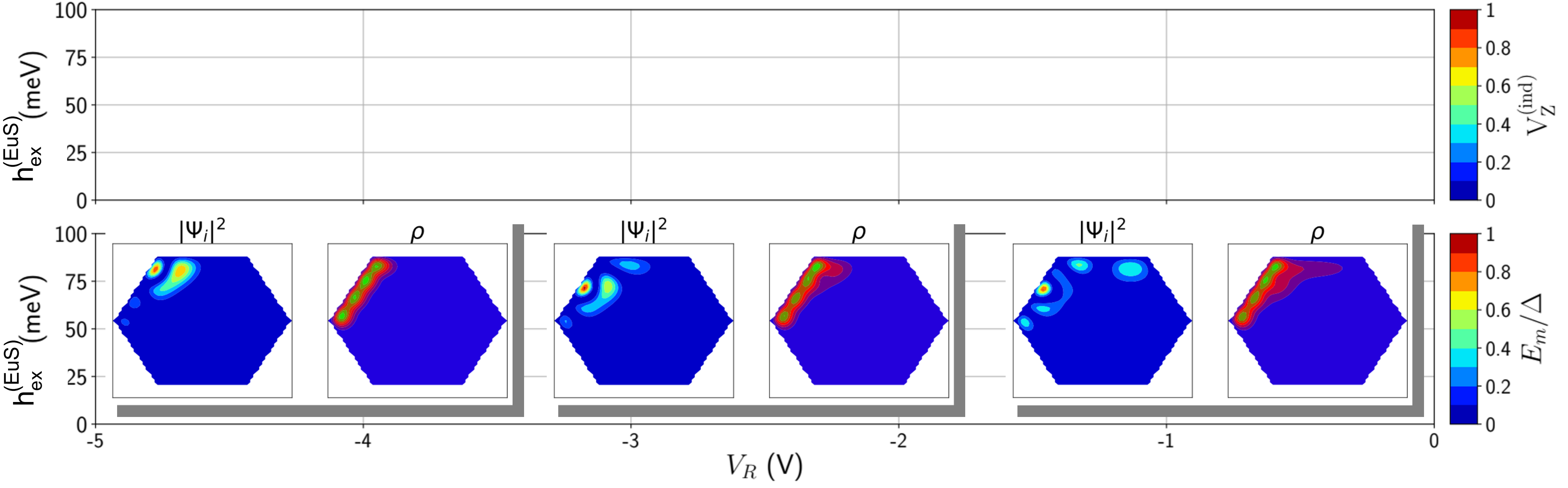
Supplementary Material

D: Extended phase diagram for the non-overlapping

Effective model

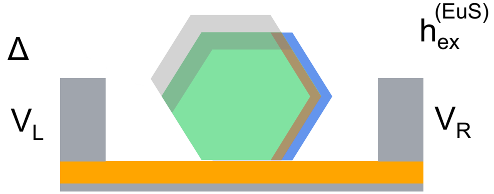
Phase diagram vs $\mathbf{V_R}$ (fixing V_L=0 and V_{bg}=-2V) for a **non-overlapping** device with direct-induced magnetization

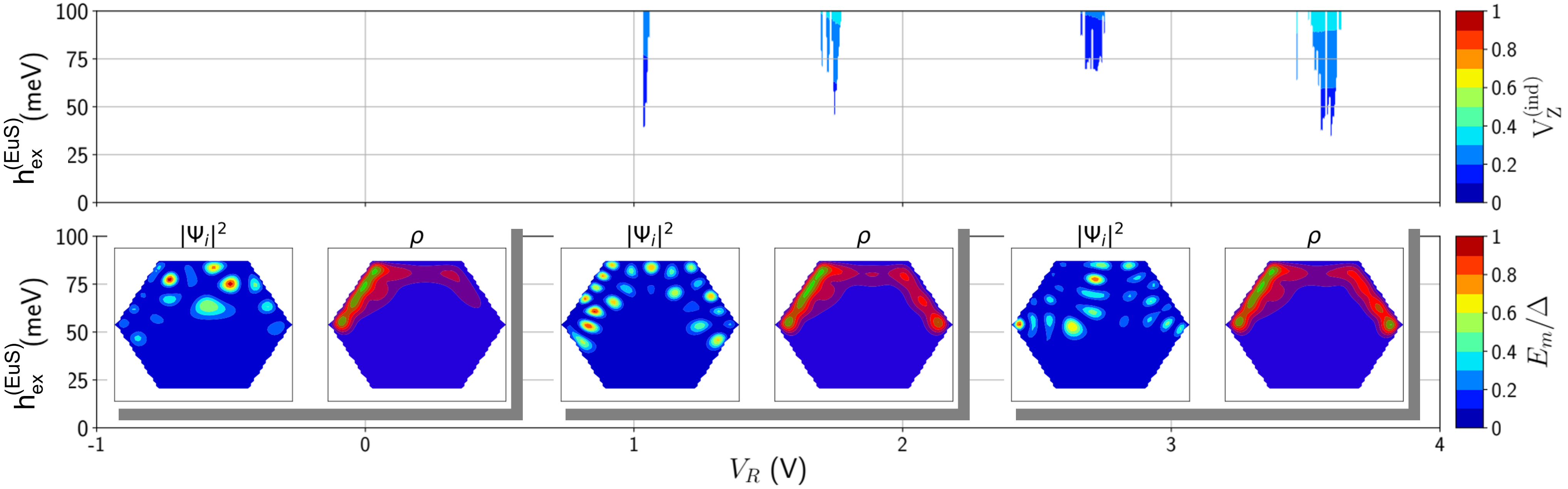




Effective model

Phase diagram vs $\mathbf{V_R}$ (fixing V_L=0 and V_{bg}=-2V) for a **non-overlapping** device with direct-induced magnetization





Effective model

Phase diagram vs $\mathbf{V_R}$ (fixing V_L=0 and V_{bg}=-2V) for a **non-overlapping** device with direct-induced magnetization