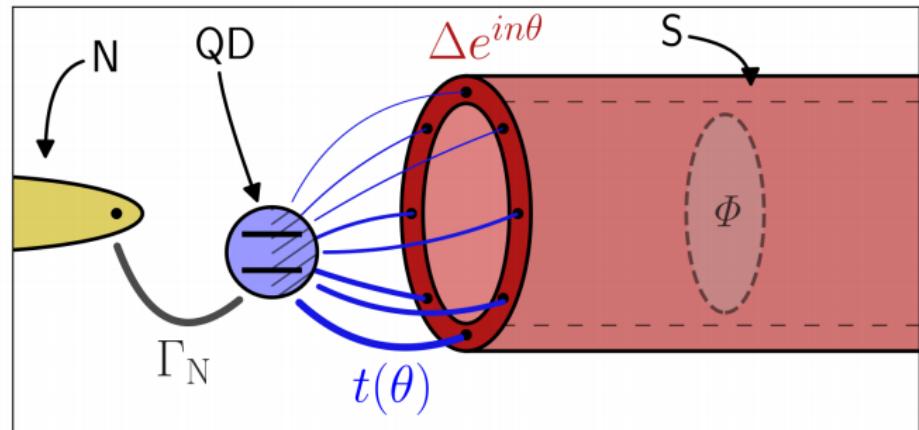


Fluxoid-induced pairing suppression and near zero-modes in quantum dots coupled to full-shell nanowires

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Theoretical Condensed Matter Physics
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Phys. Rev. B 105, 045418 (2022)
Arxiv:2107.13011

Introduction

Motivation
Model
Key observation

Co-authors



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Alfredo Levy Yeyati
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Pablo San José
ICMM-CSIC



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Introduction

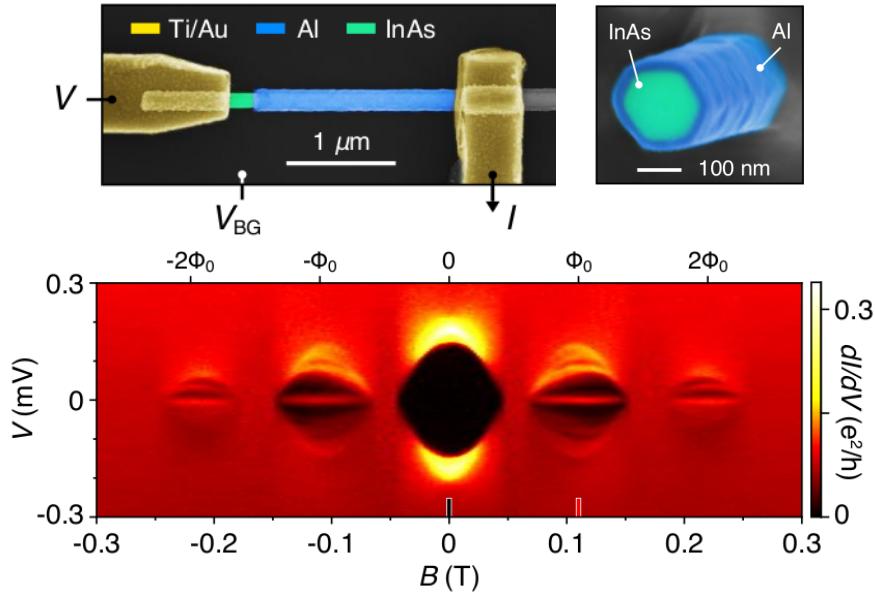
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N-QD-S with full-shell superconductors are now a possible experimental platform in semiconductor/superconductor heterostructures.

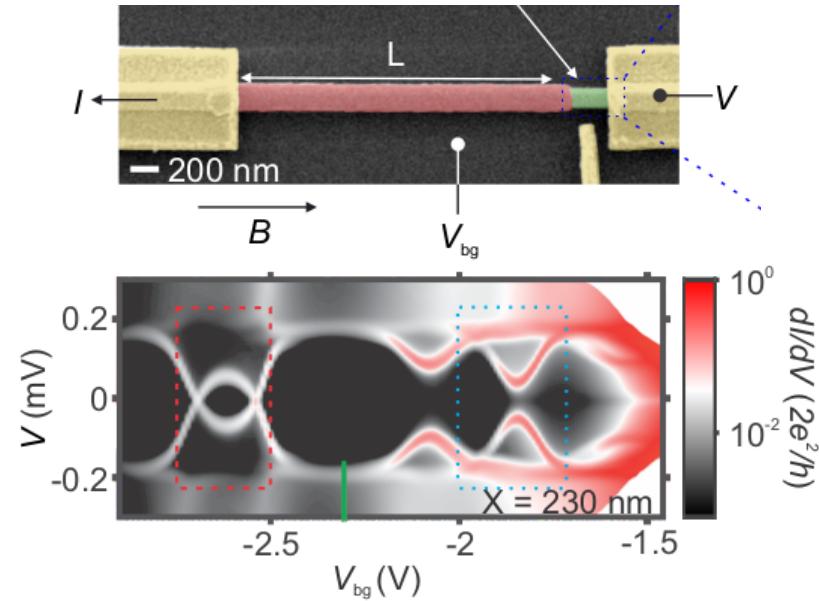
C. Marcus' group

S. Vaitiekėnas et al., *Science* 367, 1442 (2020)



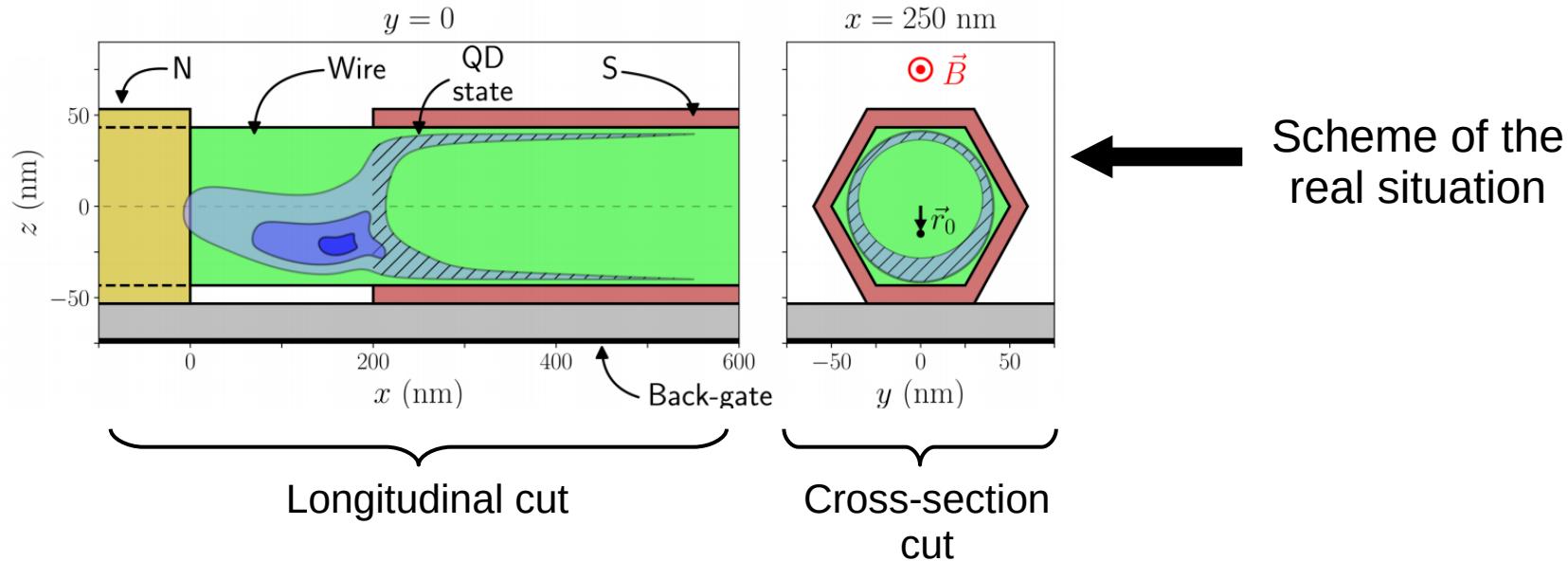
G. Katsaros' group

M. Valentini, arXiv:2008.02348 (2020)



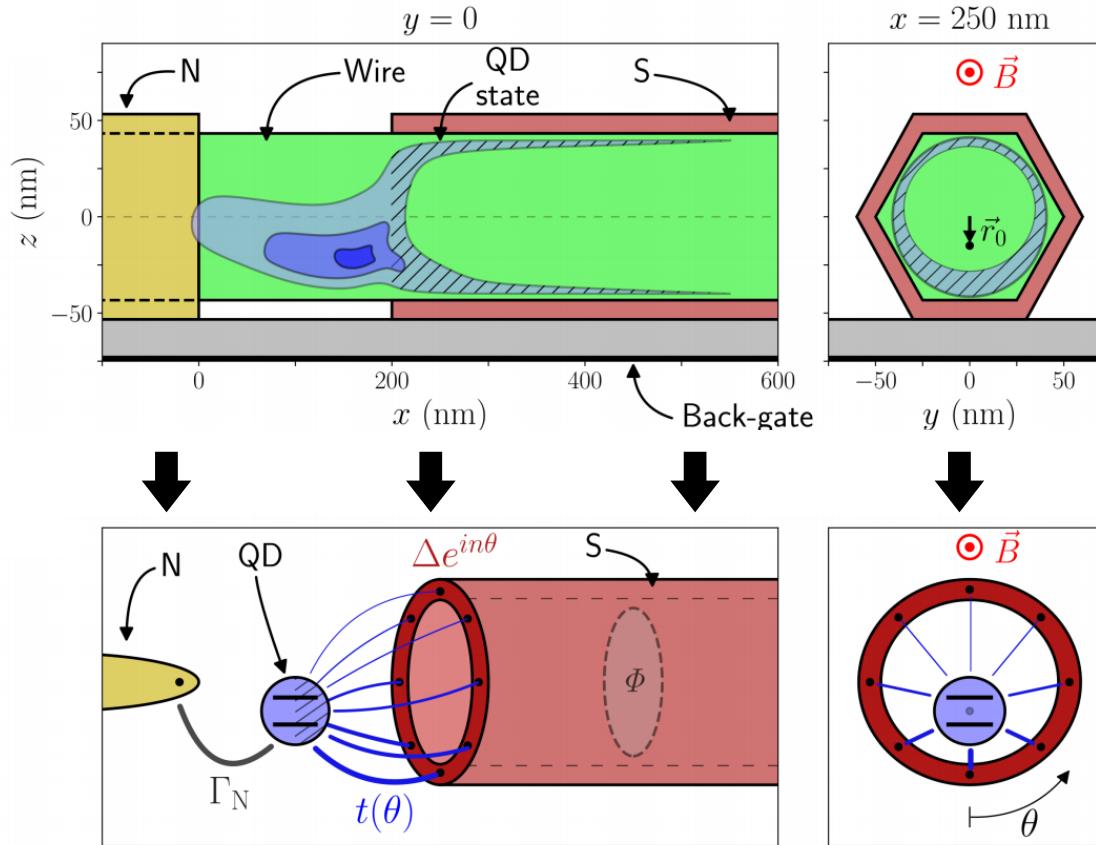
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$$\begin{aligned}
 H &= H_D + H_S + V_{SD} \\
 H_D &= \sum_{\sigma} (\epsilon_0 + \sigma V_Z) d_{\sigma}^{\dagger} d_{\sigma} + U n_{\uparrow} n_{\downarrow} \\
 H_S &= \int dz d\theta \sum_{\sigma} \left[\psi_{\sigma\theta z}^{\dagger} \frac{\mathbf{p}^2}{2m^*} \psi_{\sigma\theta z} \right. \\
 &\quad \left. + \Delta(n_{\Phi}) e^{in\theta} \psi_{\sigma\theta z}^{\dagger} \psi_{-\sigma\theta z}^{\dagger} + \text{h.c} \right] \\
 &\text{Little-Parks effects} \quad \text{Winding of the phase!} \quad n = \text{int} \left(\frac{\phi}{\phi_0} \right) \\
 V_{SD} &= \int d\theta \sum_{\sigma} t(\theta) \psi_{\sigma\theta 0}^{\dagger} d_{\sigma} + \text{h.c} \\
 t(\theta) &= t_0 \exp \left(-\alpha \frac{|\vec{r}_0 - \vec{r}_{SC}(\theta)|}{R} \right)
 \end{aligned}$$

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We write the Green's function for the QD:

$$G_{\sigma}^{QD}(\omega, \phi) = \left(\left(G_{0,\sigma}^{QD}(\omega, \phi) \right)^{-1} - \Sigma_{\sigma}^S(\omega, \phi) - \Sigma_{\sigma}^N - \Sigma_{\sigma}^{HFB} \right)^{-1}$$

- $\Sigma^{HFB} = U \begin{pmatrix} \langle n_{-\sigma} \rangle & \langle d_{\sigma} d_{-\sigma} \rangle \\ \langle d_{\sigma}^\dagger d_{-\sigma}^\dagger \rangle & -\langle n_{\sigma} \rangle \end{pmatrix}$
- $\Sigma^N = i\Gamma_N \mathbb{1}$
- Σ^S = (Large expression)

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↑
► $\Sigma^{HFB} = U \begin{pmatrix} \langle n_{-\sigma} \rangle & \langle d_{\sigma} d_{-\sigma} \rangle \\ \langle d_{\sigma}^{\dagger} d_{-\sigma}^{\dagger} \rangle & -\langle n_{\sigma} \rangle \end{pmatrix}$

► $\Sigma^N = i\Gamma_N \mathbb{1}$

► $\Sigma^S \approx -\frac{\Gamma_S}{\sqrt{\Delta^2 - \omega^2}} \begin{pmatrix} \omega & \Delta\delta_n \\ \Delta\delta_n & \omega \end{pmatrix}$

← Axial symmetric case $\begin{cases} \vec{r}_0 = (0, 0) \\ t(\theta) = \text{cte.} \end{cases}$

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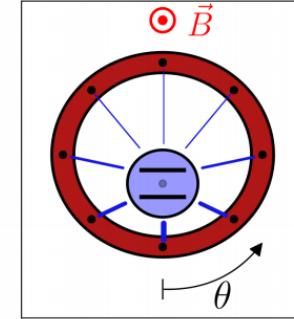
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- ▶ $\Sigma^S \approx -\frac{\Gamma_S}{\sqrt{\Delta^2 - \omega^2}} \begin{pmatrix} \omega & \Delta\delta_n \\ \Delta\delta_n & \omega \end{pmatrix}$

$$\Sigma^S = \int d\theta d\bar{\theta} t(\theta + \bar{\theta}) g_S(\theta + \bar{\theta}, \theta - \bar{\theta}) t(\theta - \bar{\theta})$$

$$g_S \sim \begin{pmatrix} \omega & \Delta e^{in\theta} \\ \Delta e^{-in\theta} & \omega \end{pmatrix}$$

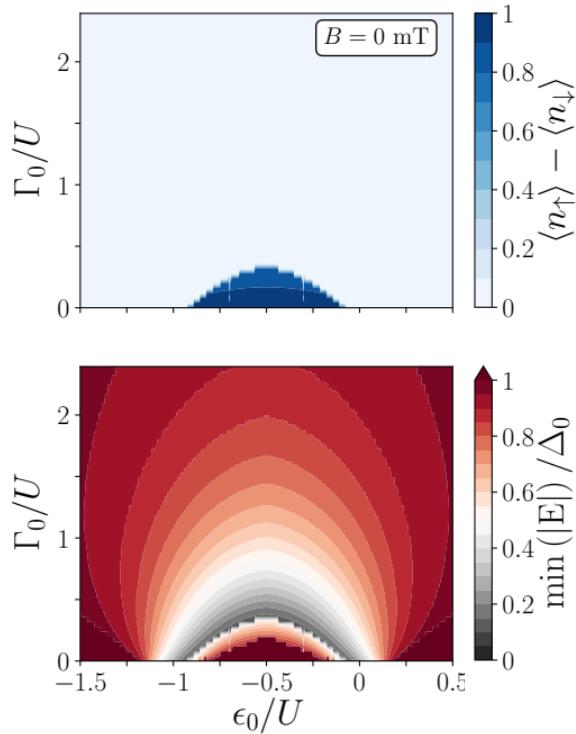


**Pairing term is zero for $n>0$
when the integral is done!
(in the symmetric coupling)**

Results

LDOS versus QD energy level
LDOS versus magnetic field
Electrostatic simulations

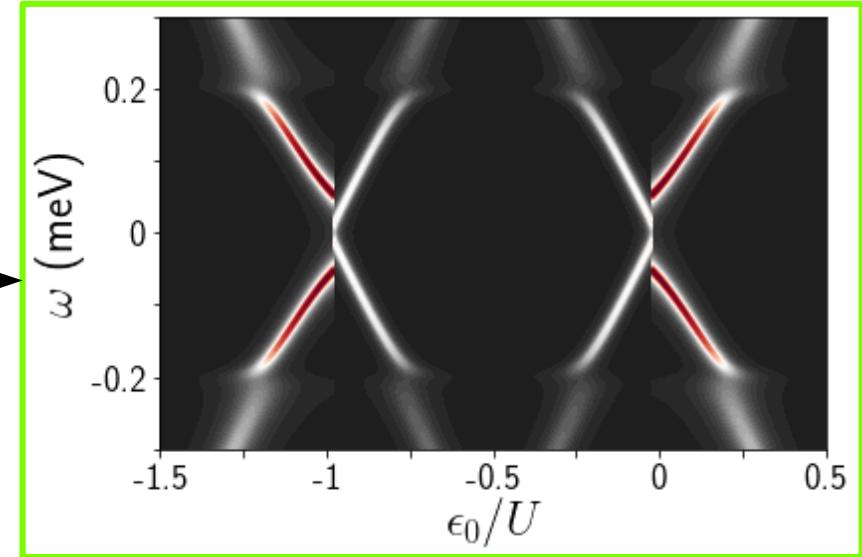
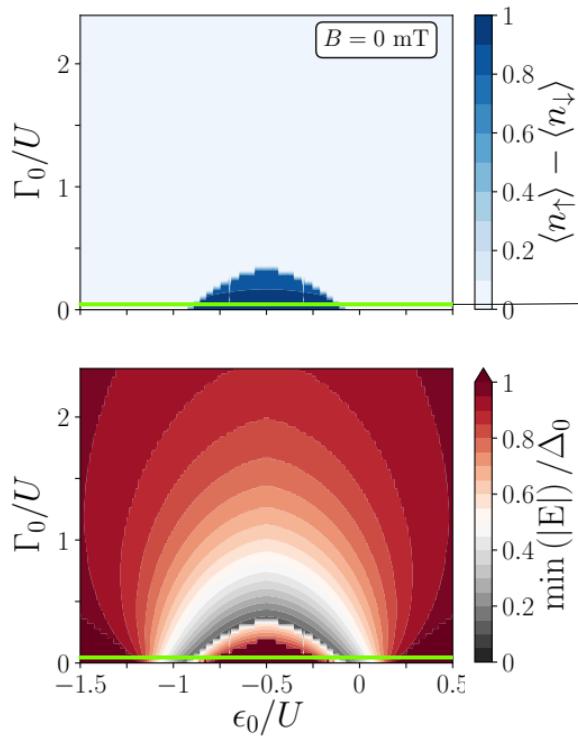
n=0
Symmetric case



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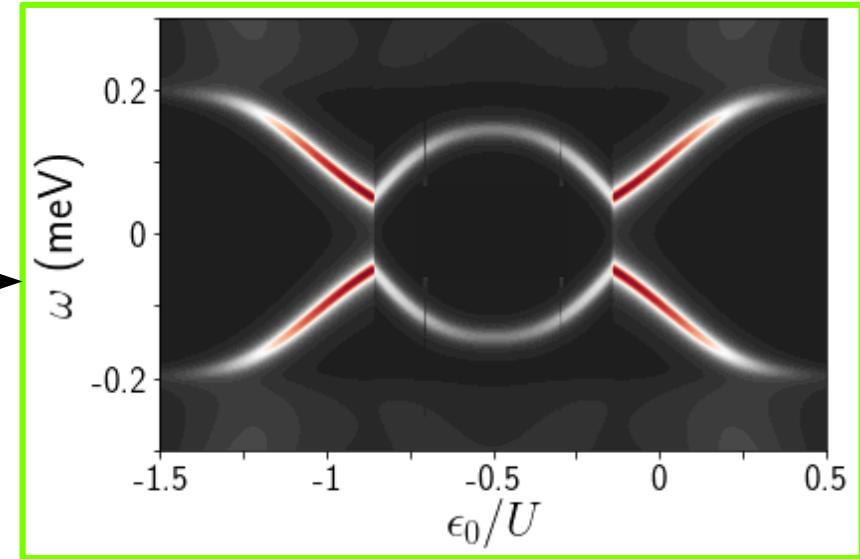
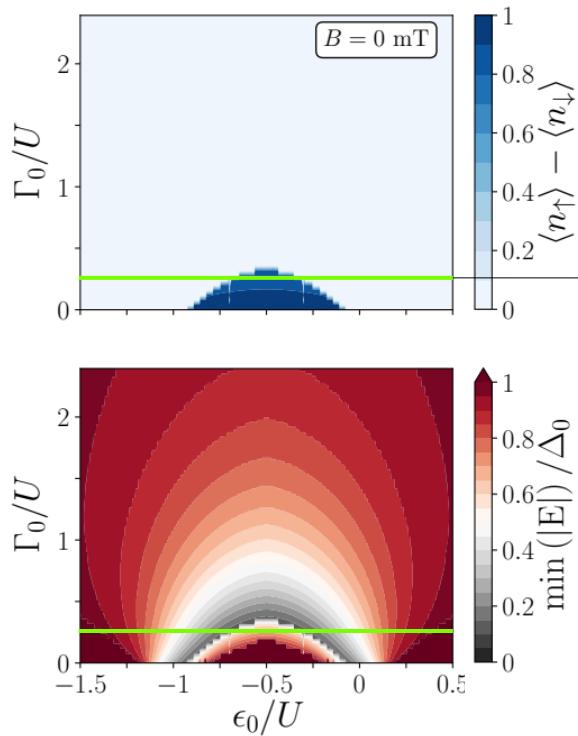
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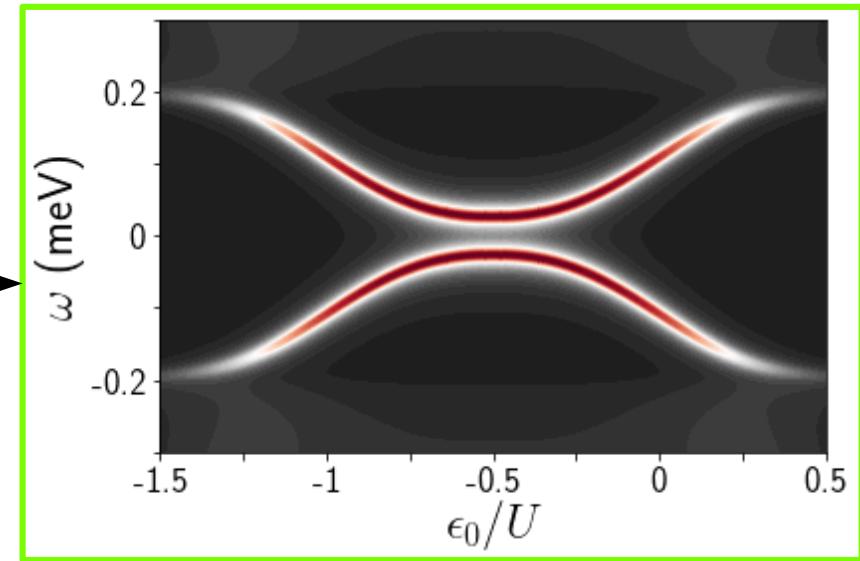
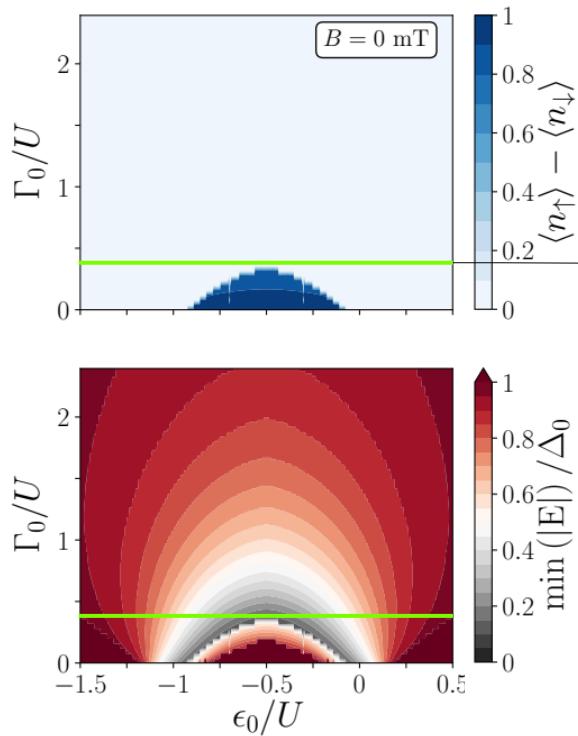
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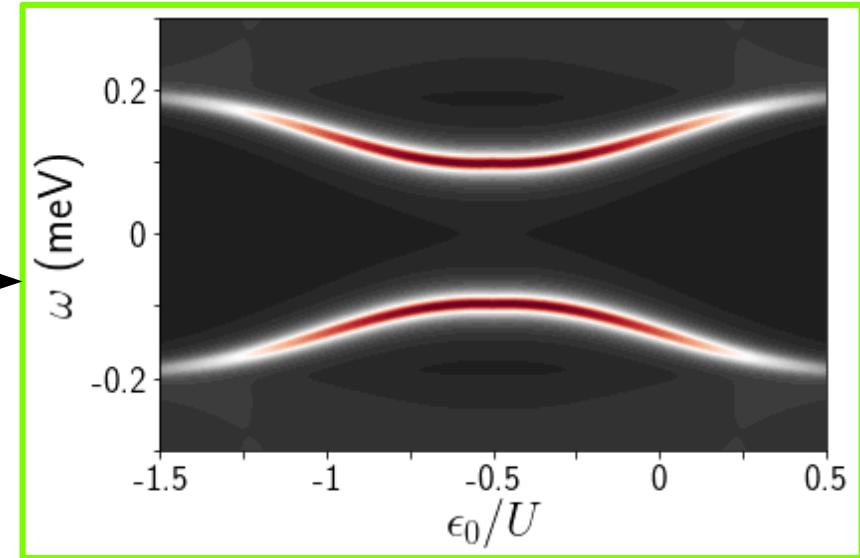
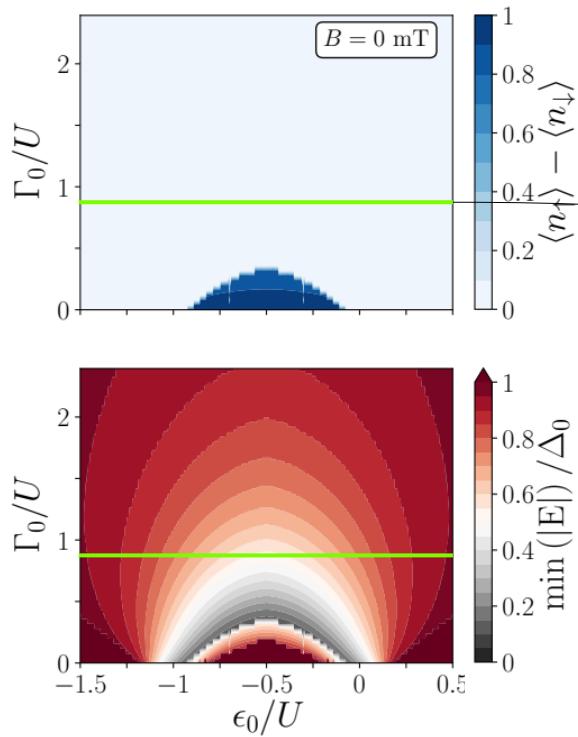
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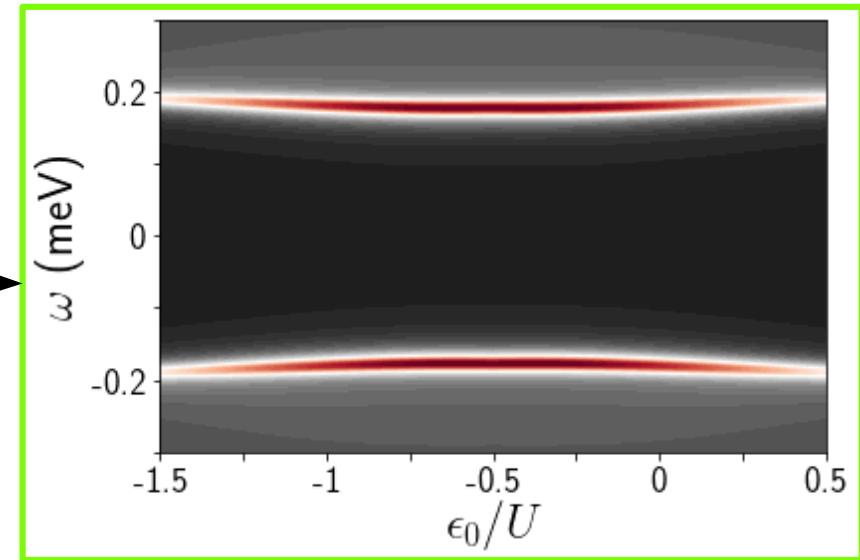
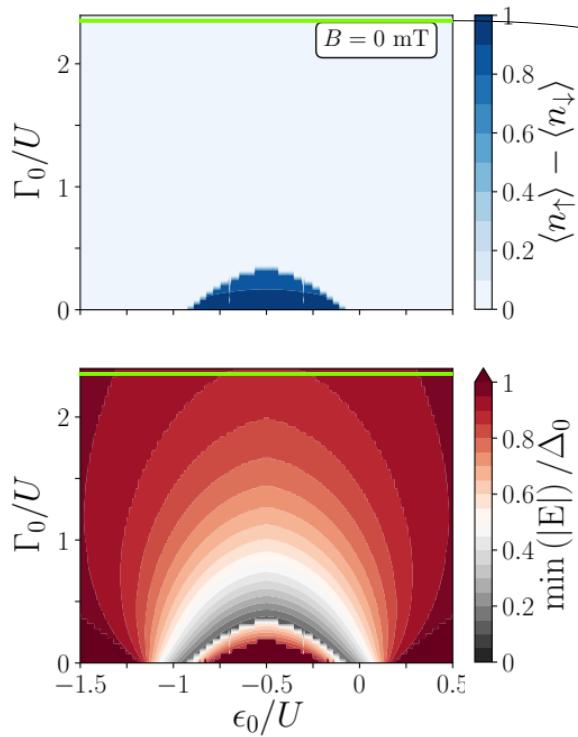
n=0
Symmetric case



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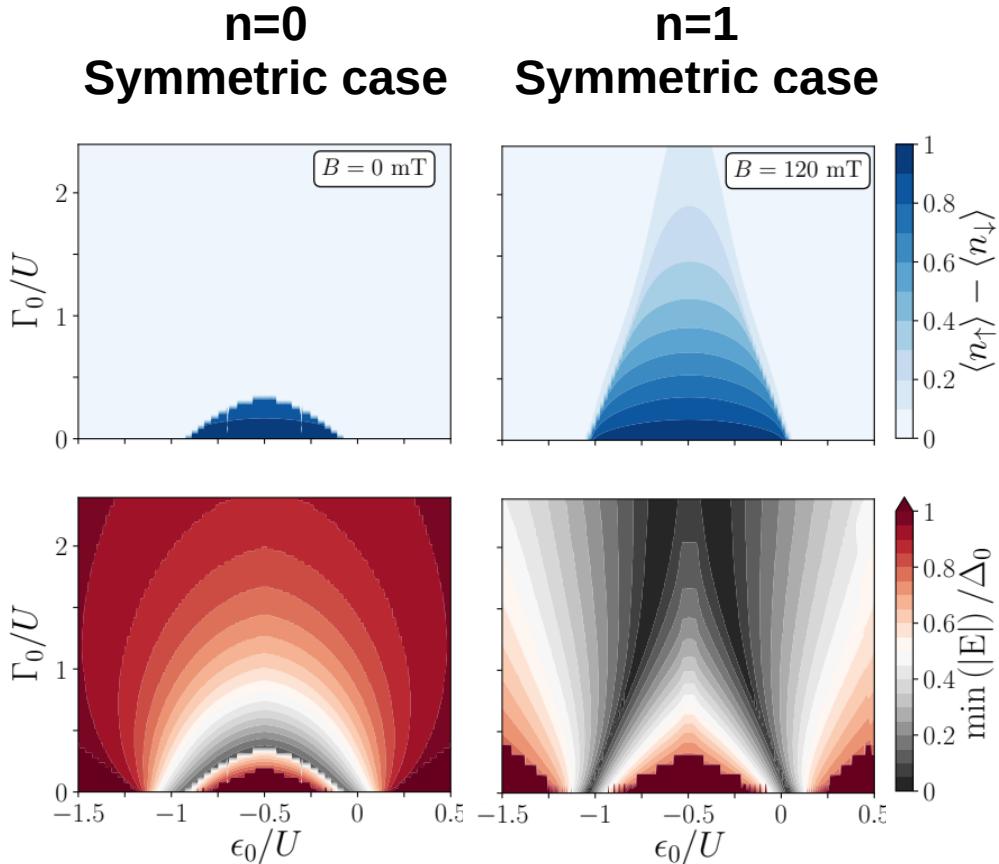
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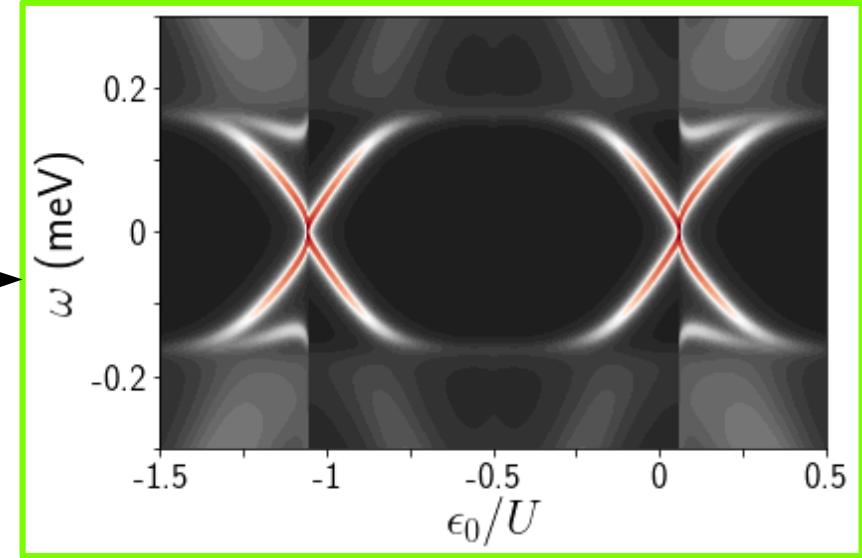
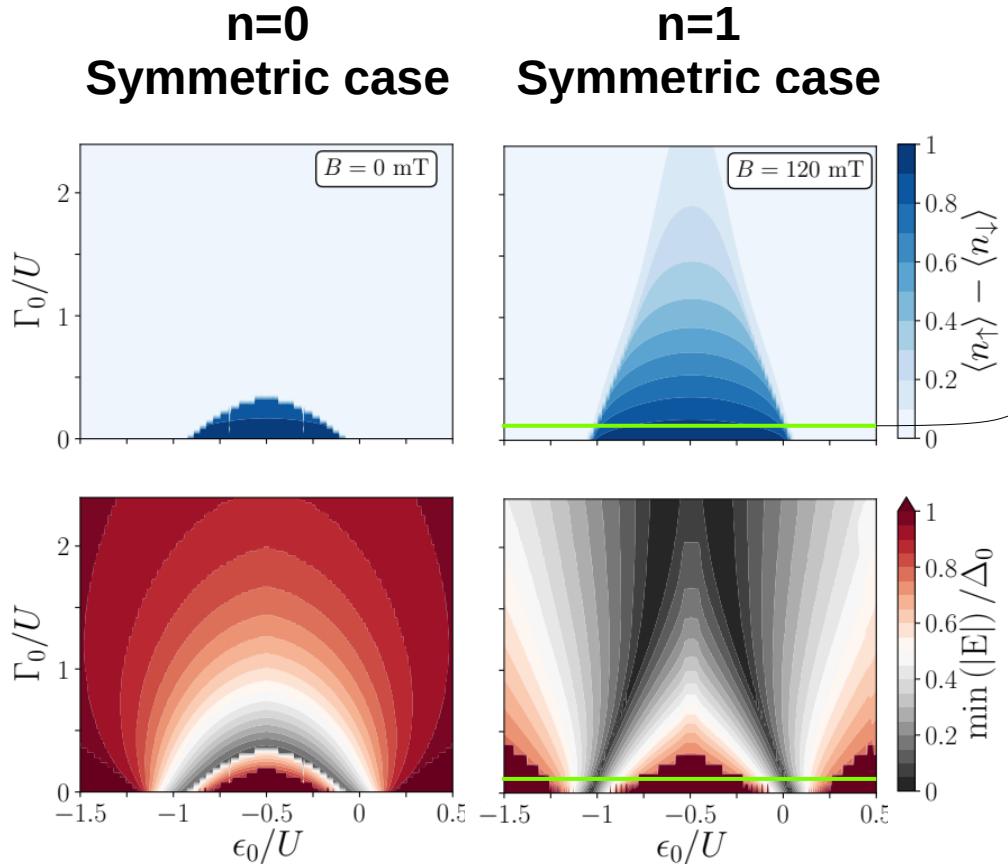
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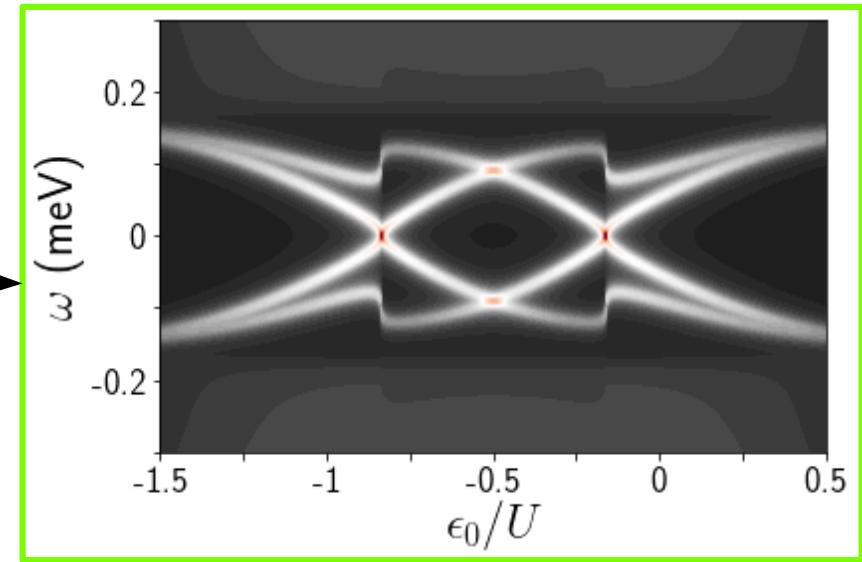
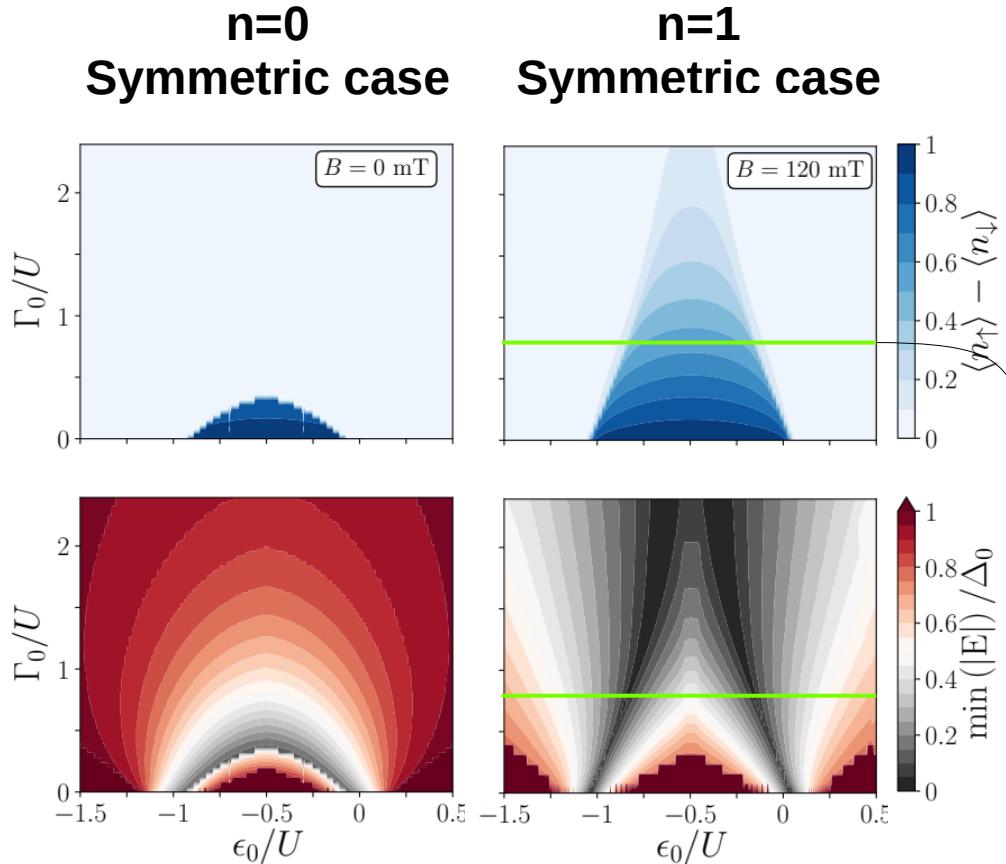
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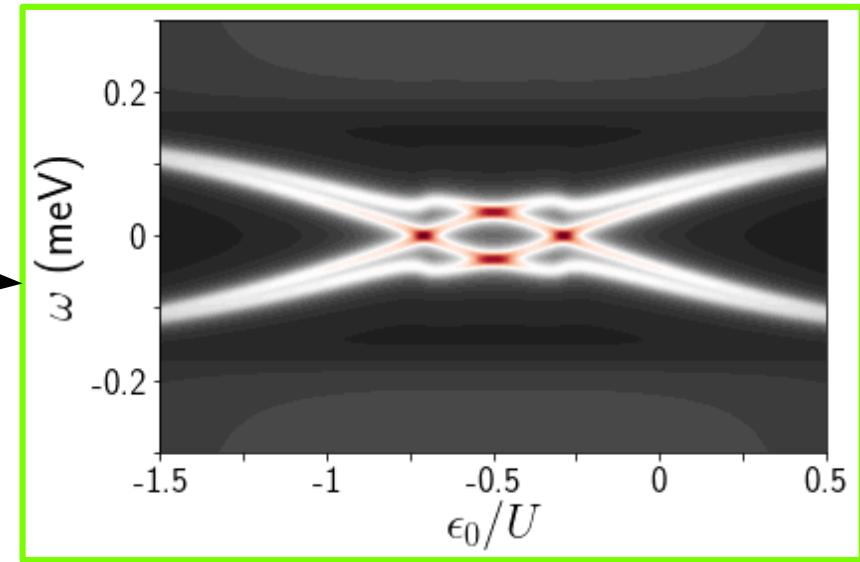
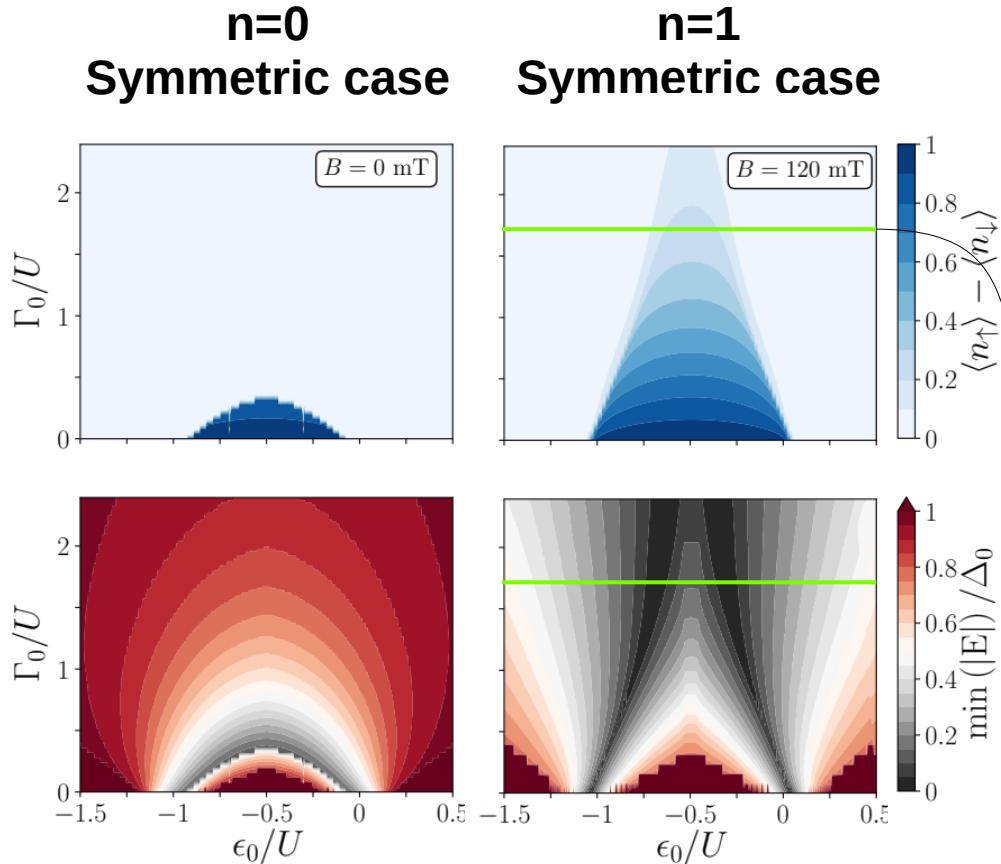
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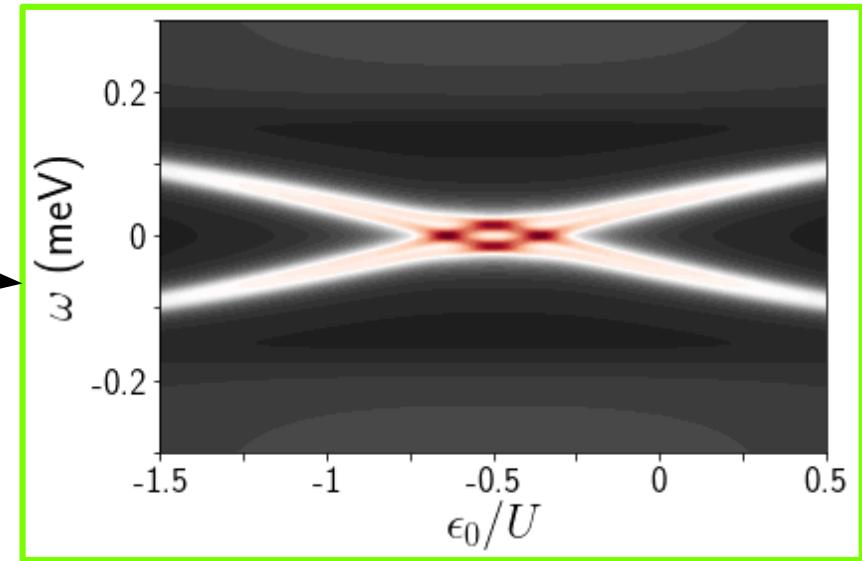
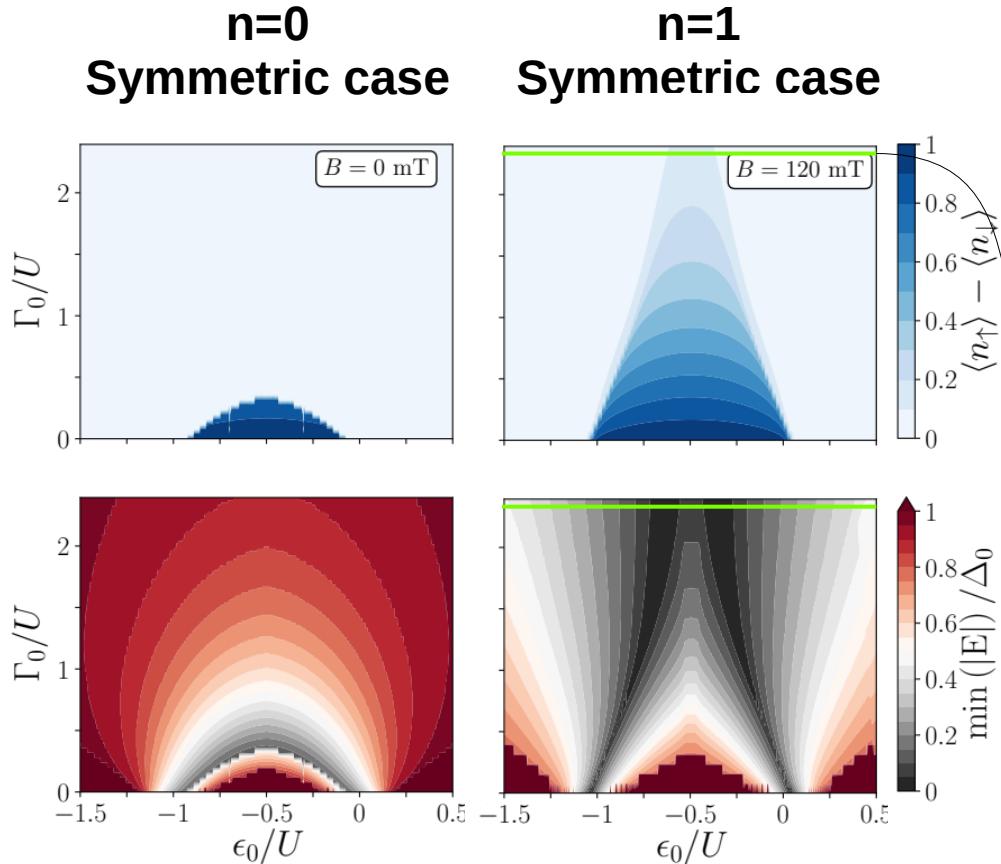
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Results

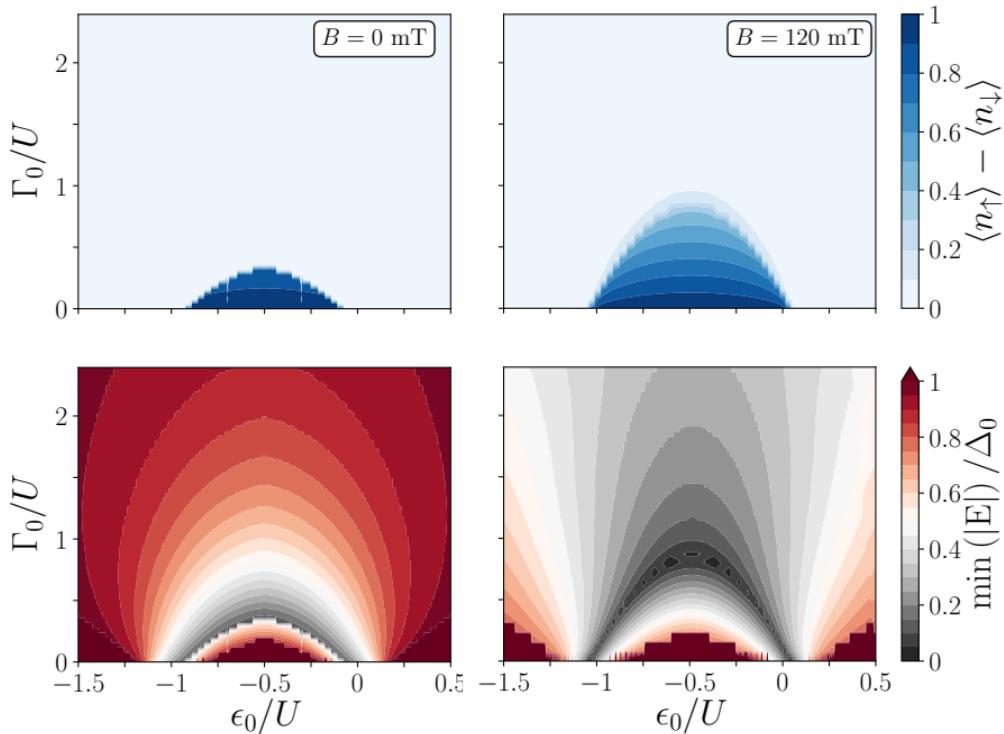
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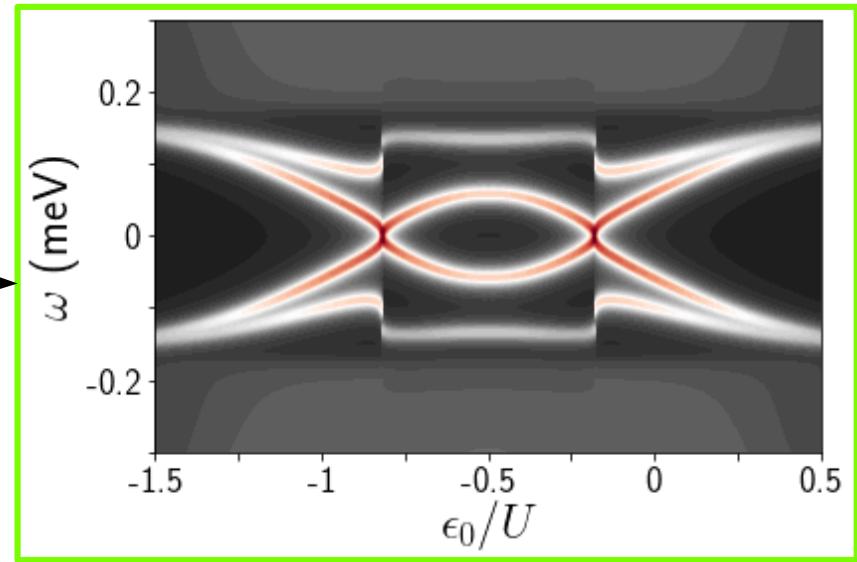
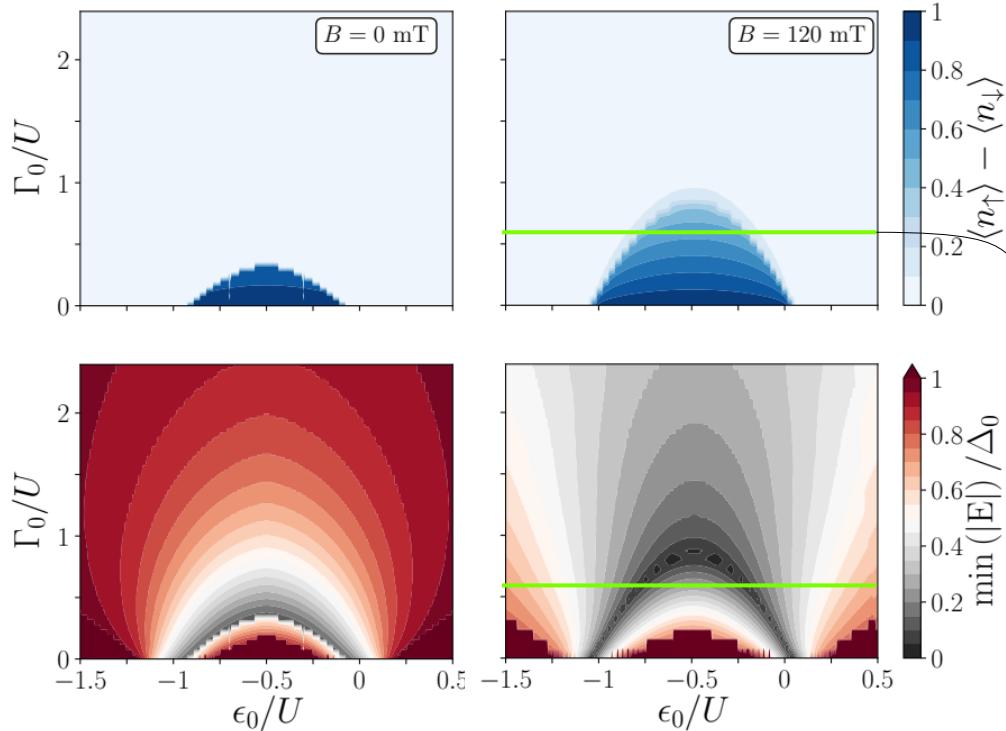
n=0
Asymmetric case **n=1**
Asymmetric case



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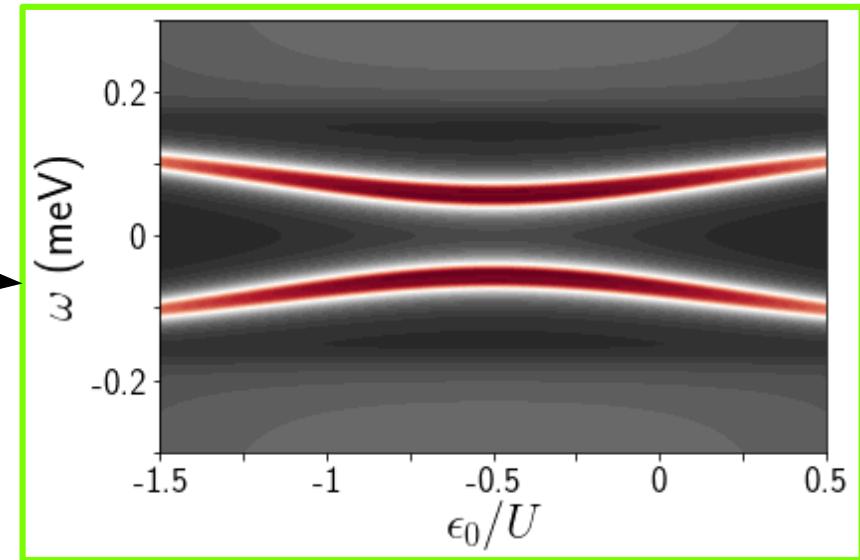
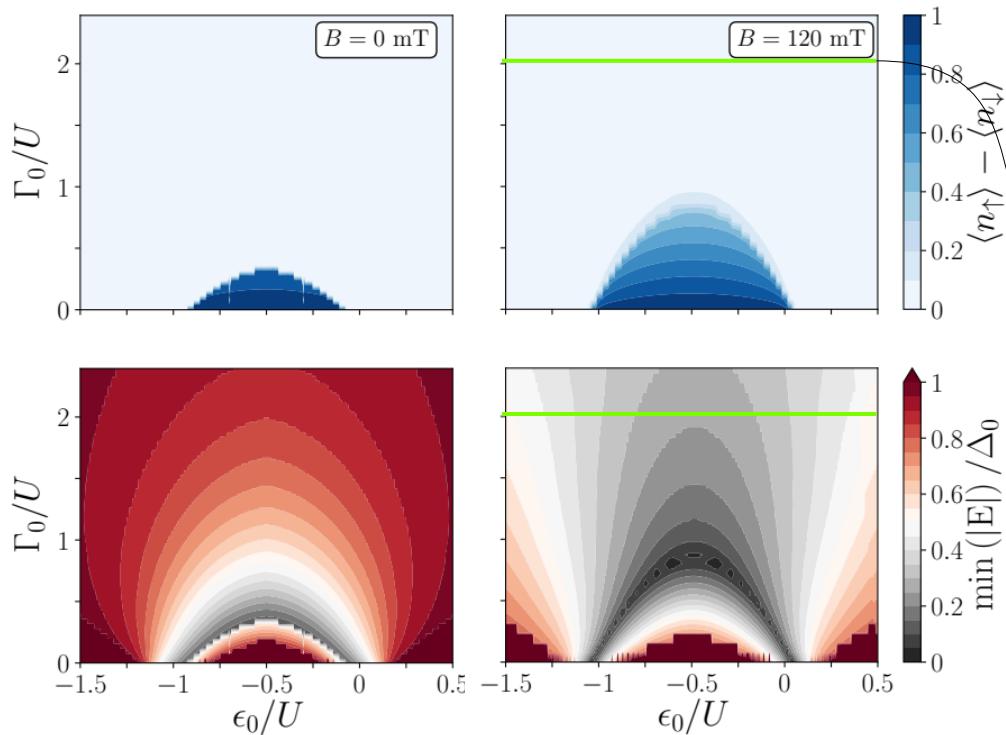
n=0
Asymmetric case **n=1**
Asymmetric case



Results

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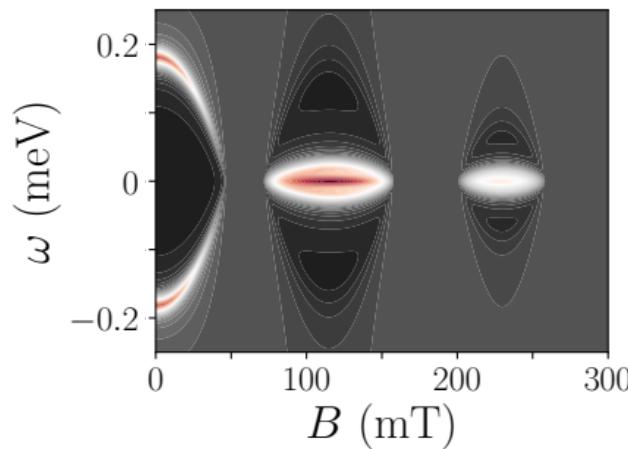
n=0
Asymmetric case **n=1**
Asymmetric case



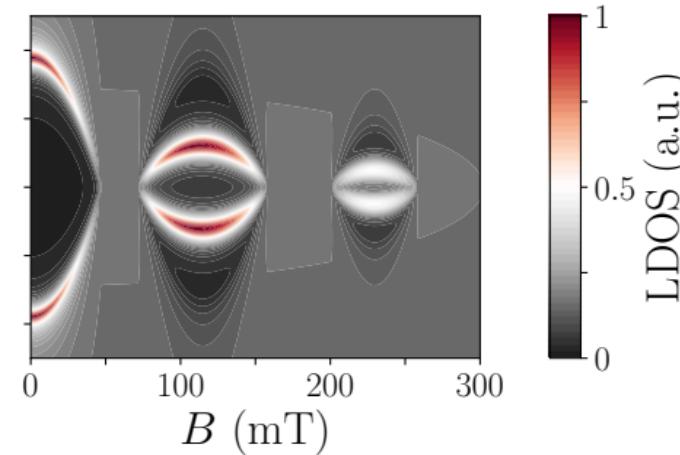
Results

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**Strong coupling,
Symmetric case**



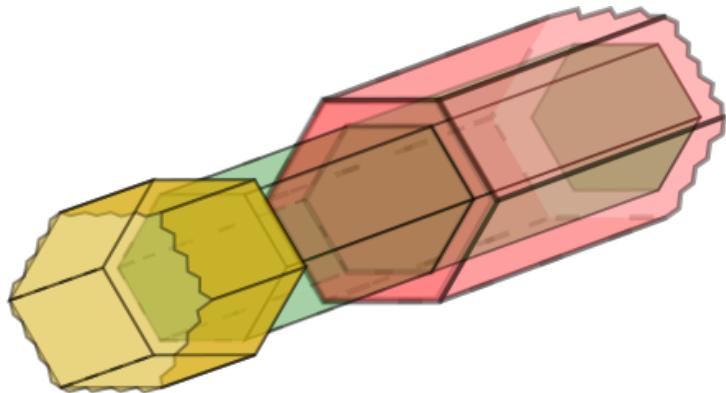
**Strong coupling,
Asymmetric case**



**QD states may give rise to similar features to those
of Majorana Bound States in full-shell wires**

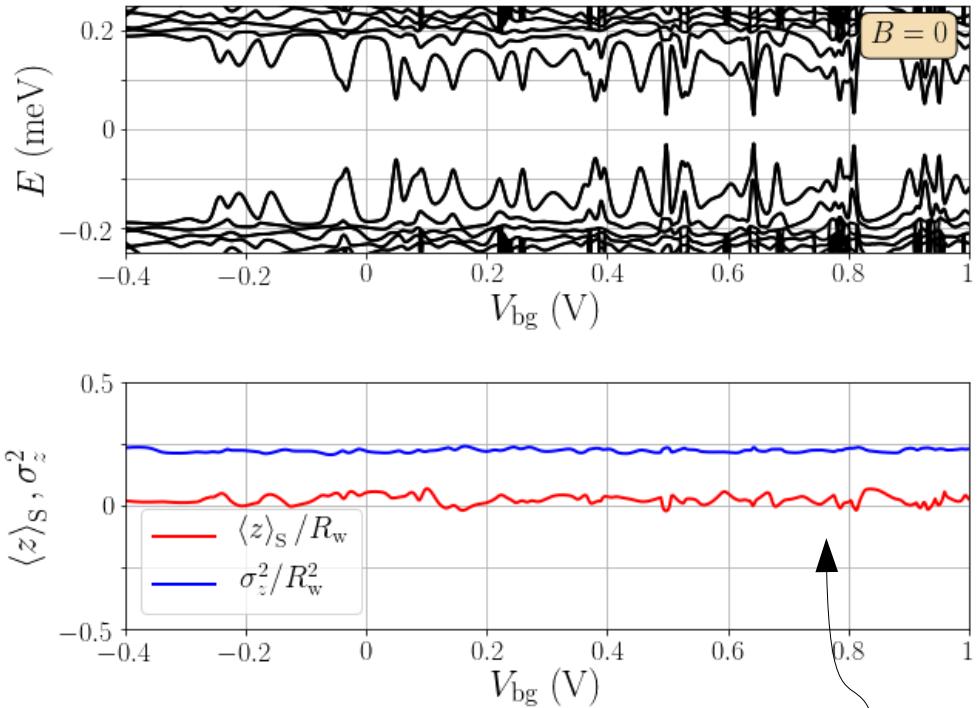
Results

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$$H = \left(\frac{\hbar^2 \vec{k}^2}{2m^*} + e\phi(\vec{r}) \right) \sigma_0 \tau_z + \Delta(\vec{r}) \sigma_y \tau_y$$
$$+ \frac{1}{2} \left[\vec{\alpha}(\vec{r}) \cdot (\vec{\sigma} \times \vec{k}) + (\vec{\sigma} \times \vec{k}) \cdot \vec{\alpha}(\vec{r}) \right] \tau_z$$

Full Schrödinger-Poisson simulations

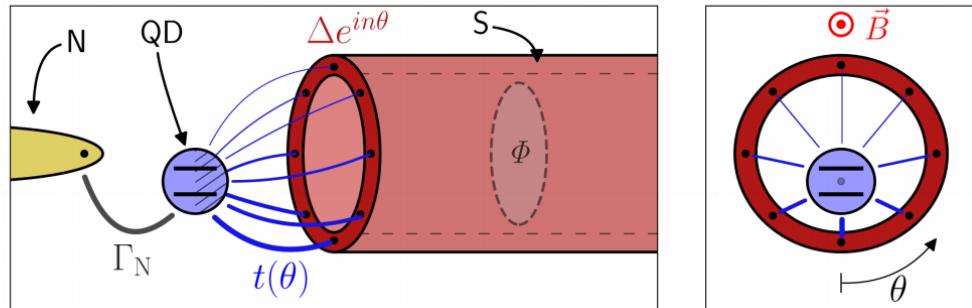


Symmetric case is a realistic situation!

Conclusions

Take-home messages

- N-QD-S junctions with full-shell superconductors develop quite different features and phase diagrams, specially in some cases.
- In the axial symmetric case, the superconducting pairing is always suppressed for the $n>0$ lobes.
- The YSR states are further pushed towards zero energy compared to a conventional N-QD-S junction.



Reference

[Arxiv:2107.13011](https://arxiv.org/abs/2107.13011)

**Thank you for
your attention!**