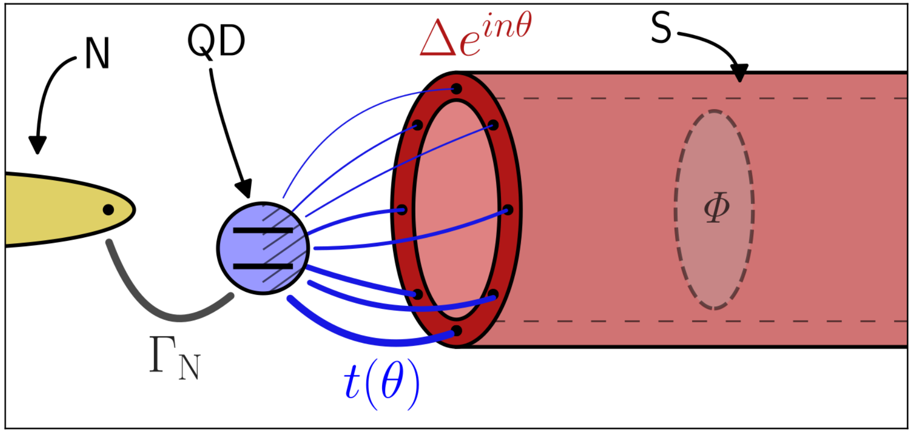
Vortex-induced pairing suppression and near zero-modes in quantum dots coupled to full-shell nanowires

Samuel D. Escribano

Theoretical Condensed Matter Physics Department

Universidad Autónoma de Madrid



Co-authors

Elsa Prada
ICMM-CSIC

Pablo San José
ICMM-CSIC

Ramón Aguado
ICMM-CSIC

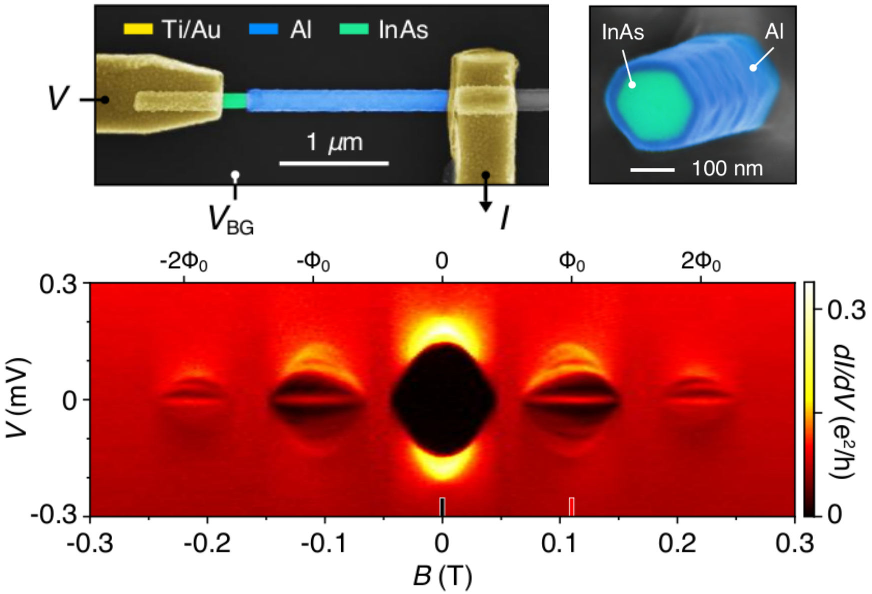
Alfredo Levy Yeyati
UAM

Introduction

N-QD-S with full-shell superconductors are now a possible experimental platform in semiconductor/superconductor heterostrucures.

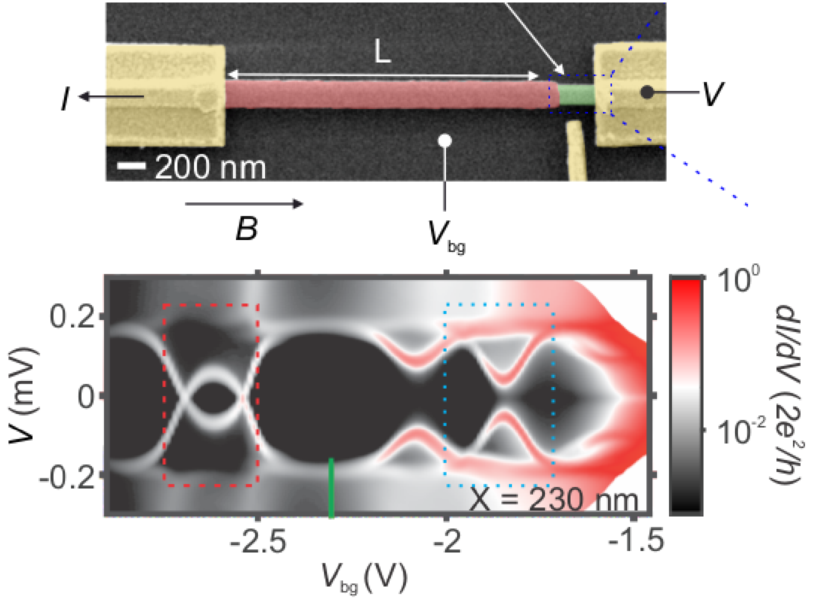
C. Marcus' group

S. Vaitiekenas et al., Science 367, 1442 (2020)

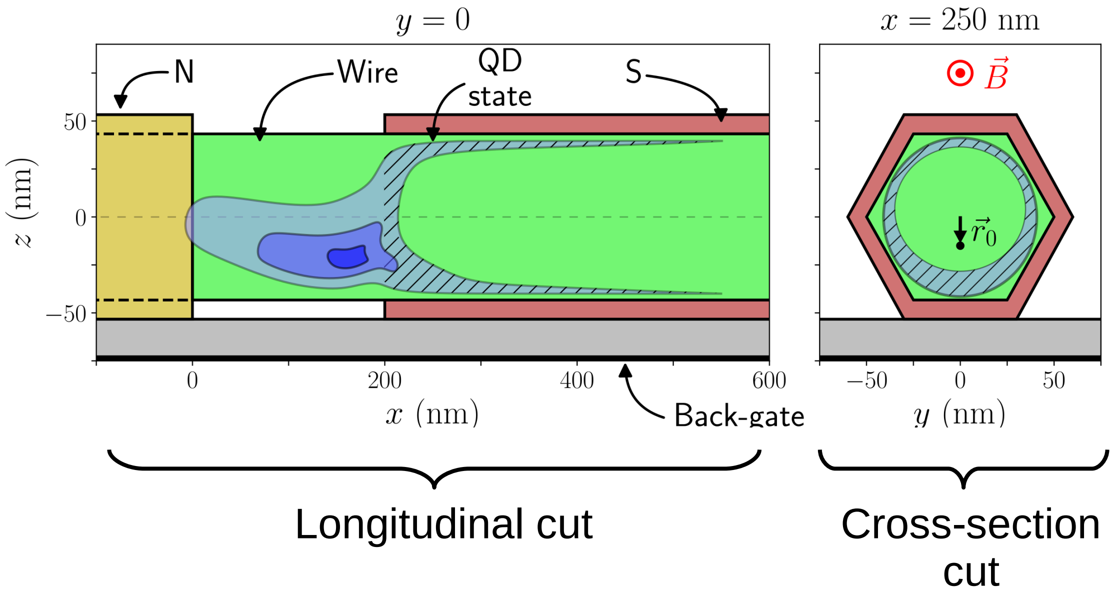


G. Katsaros' group

M. Valentini, arXiv:2008.02348 (2020)

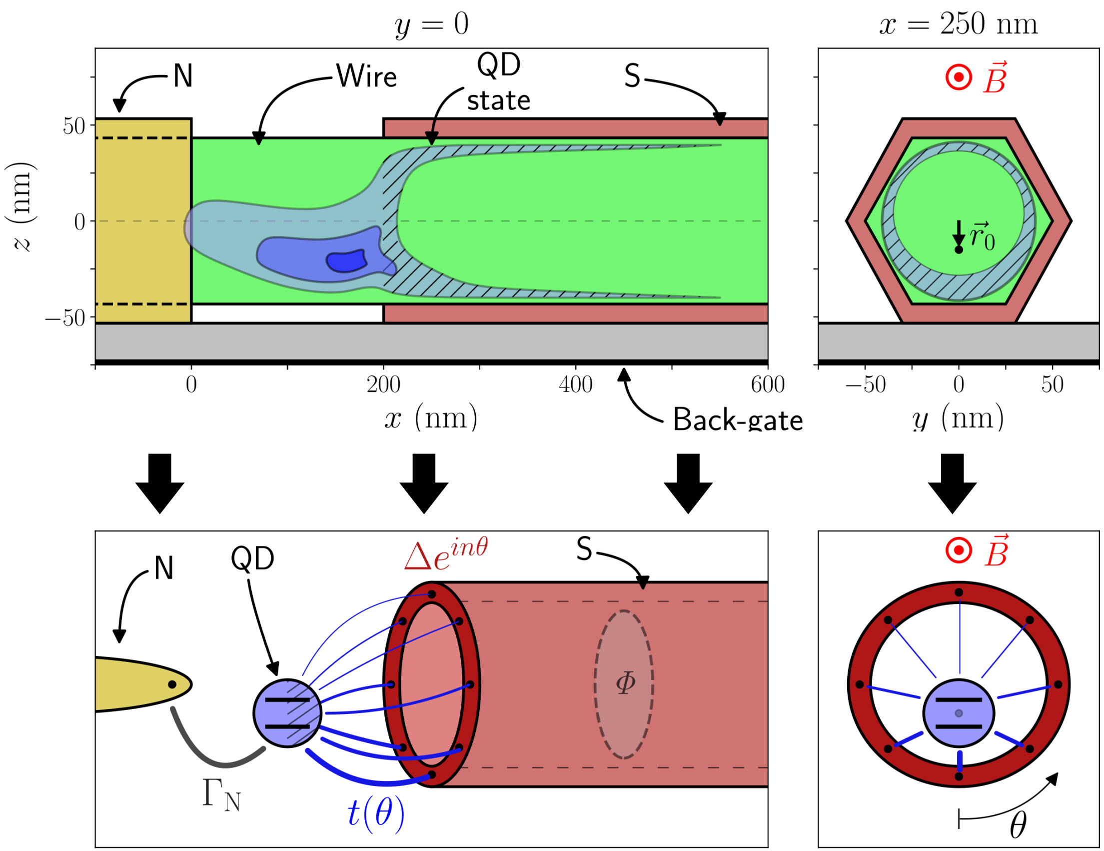


Introduction



Scheme of the real situation

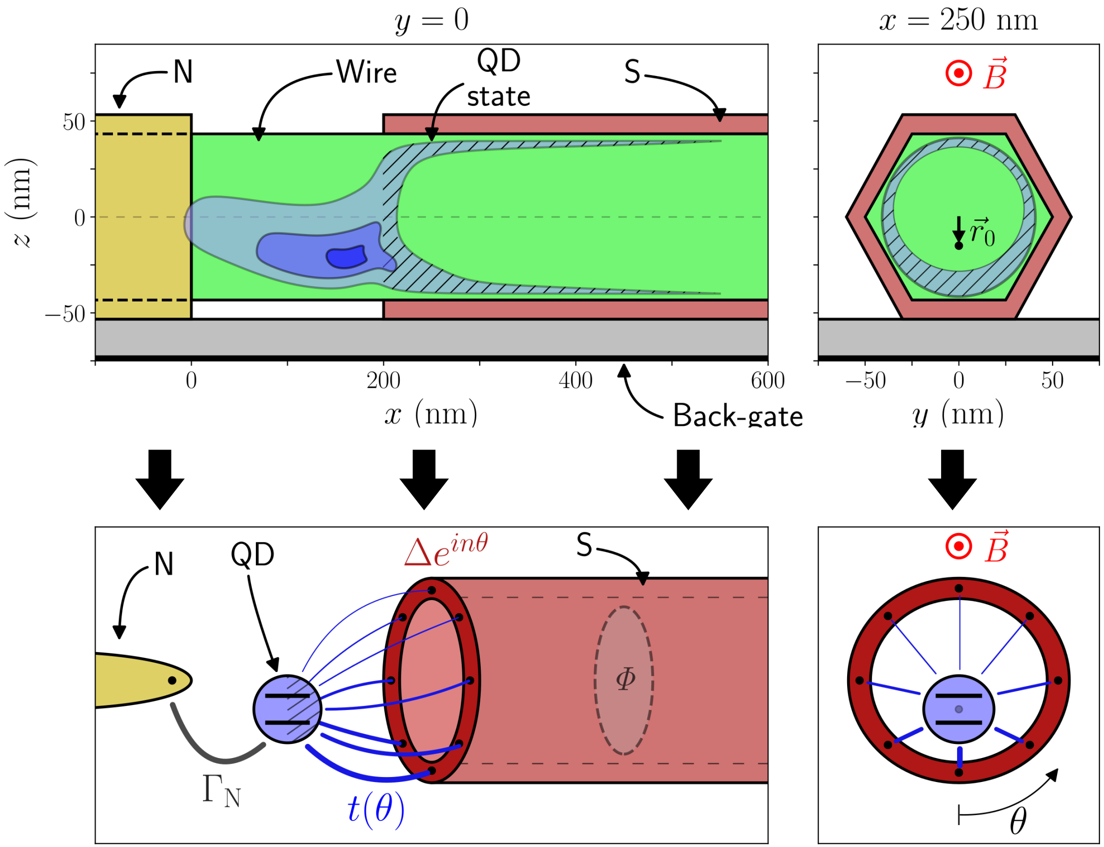
Introduction



Scheme of the real situation

Our N-QD-S model

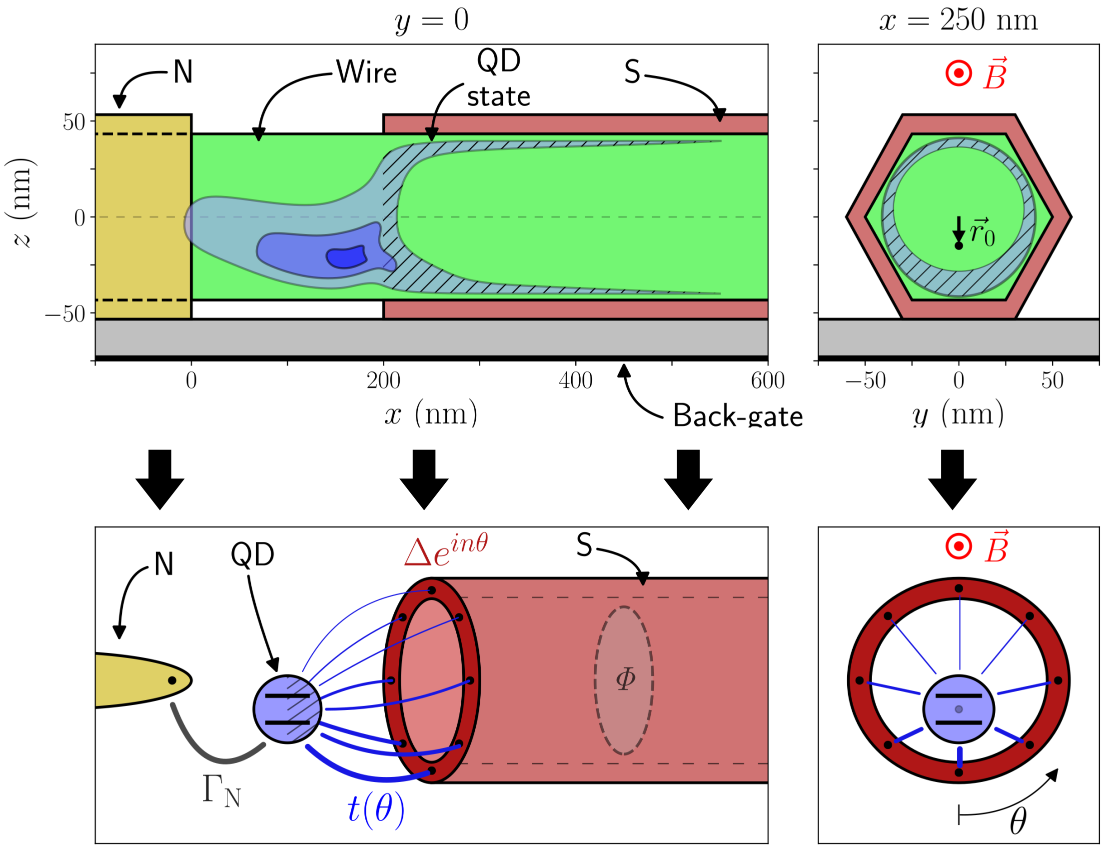
Introduction



$$H = H_D + H_S + V_{SD}$$

We extend the Anderson Model to this system.

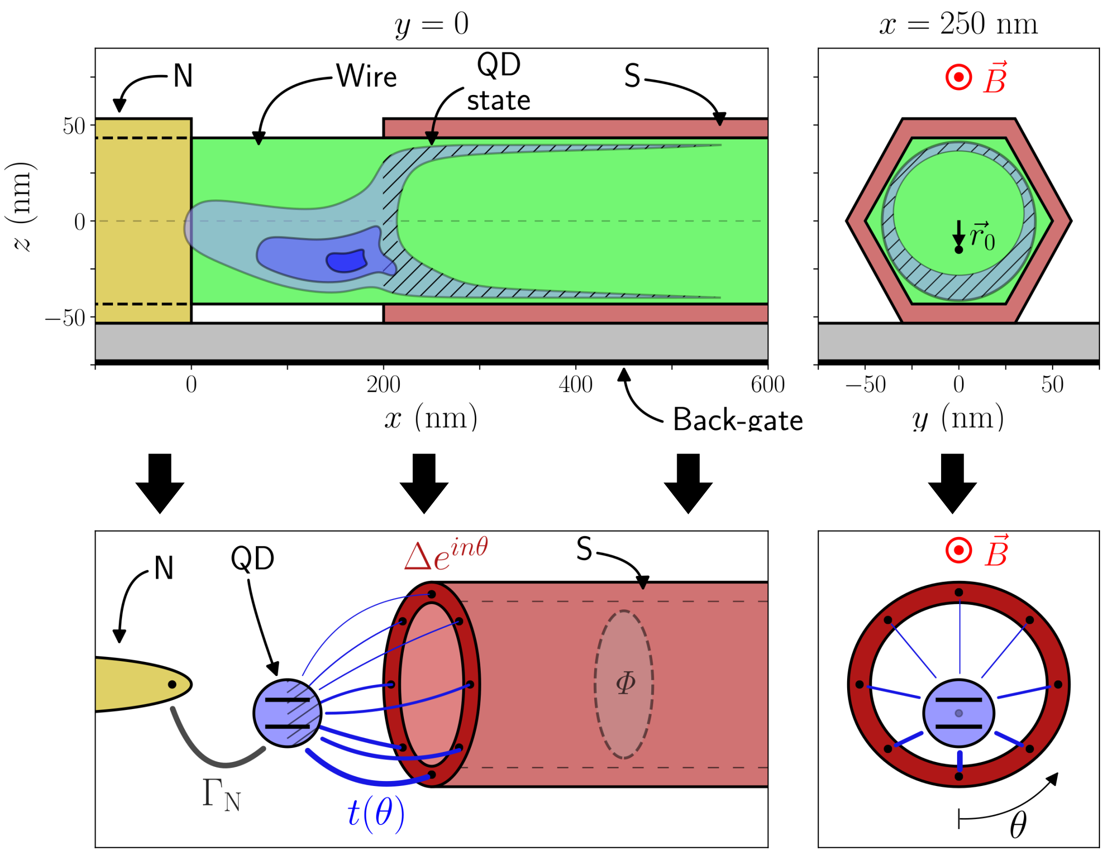
Introduction



$$H = H_D + H_S + V_{SD}$$

$$H_D = \sum_\sigma (\epsilon_0 + \sigma V_Z) d_\sigma^\dagger d_\sigma + U n_\uparrow n_\downarrow$$

We treat it at mean-field level (but our main conclusions are general).



$$H = H_D + H_S + V_{SD}$$

$$H_D = \sum_\sigma (\epsilon_0 + \sigma V_Z) d^\dagger_\sigma d_\sigma + U n_\uparrow n_\downarrow$$

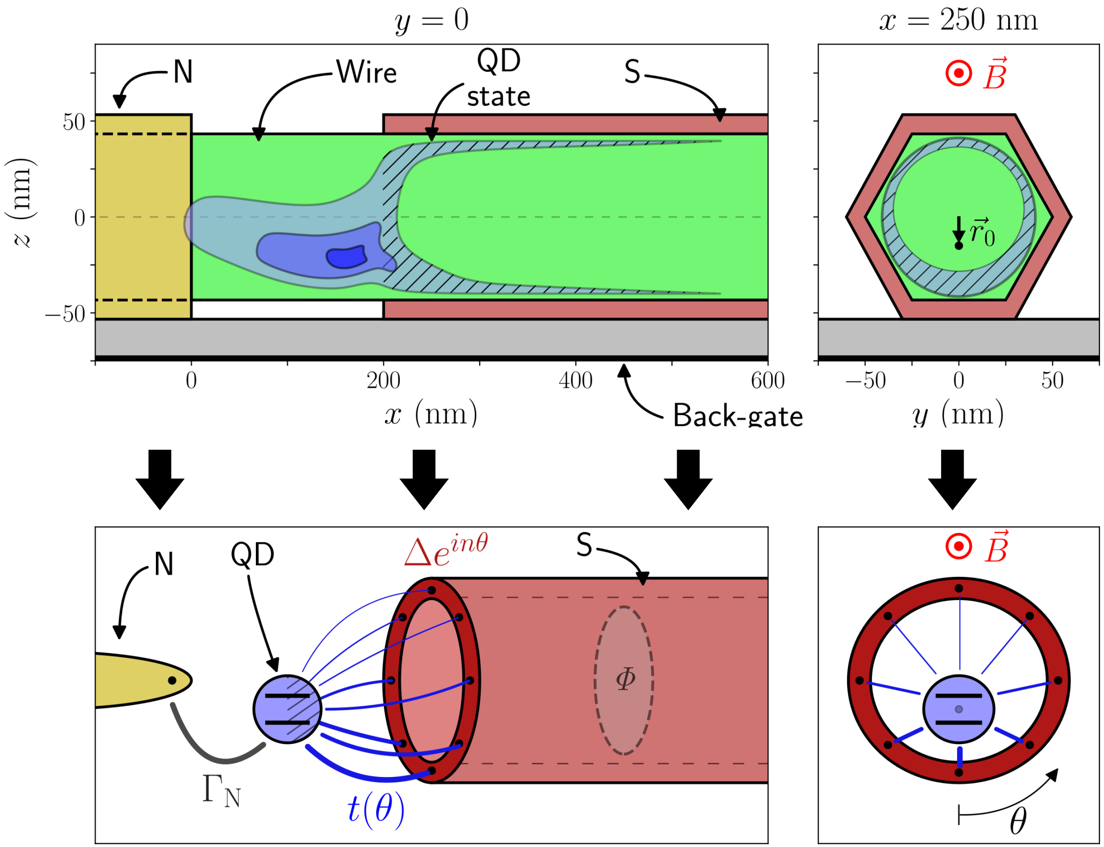
$$H_S = \int dz\, d\theta \sum_\sigma \left[\psi^\dagger_{\sigma\theta z} \frac{\boldsymbol{p}^2}{2m^*} \psi_{\sigma\theta z} + \Delta(n_\Phi) e^{in\theta} \psi^\dagger_{\sigma\theta z} \psi^\dagger_{-\sigma\theta z} + \text{h.c} \right]$$

Little-Parks effects

Winding of the phase!

$$n = \text{int}\left(\frac{\phi}{\phi_0}\right)$$

Introduction



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$$V_{SD} = \int d\theta \sum_\sigma t(\theta) \psi^\dagger_{\sigma\theta 0} d_\sigma + \text{h.c}$$

$$t(\theta) = t_0 \exp\left(-\alpha \frac{|\vec{r}_0 - r_{\text{SC}}(\theta)|}{R} \right)$$

Introduction

We write the Green's function for the QD:

$$G_\sigma^{QD}(\omega,\phi) = \left(\left(G_{0,\sigma}^{QD}(\omega,\phi) \right)^{-1} - \Sigma_\sigma^S(\omega,\phi) - \Sigma_\sigma^N - \Sigma_\sigma^{HFB} \right)^{-1}$$

$$\Sigma^{\mathrm{HFB}} = U \begin{pmatrix} \langle n_{-\sigma} \rangle & \langle d_\sigma d_{-\sigma} \rangle \\ \left\langle d_\sigma^\dagger d_{-\sigma}^\dagger \right\rangle & -\langle n_\sigma \rangle \end{pmatrix}$$

$$\Sigma^N = i\Gamma_N \mathbb{1}$$

Σ^S = (Large expression)

We write the Green's function for the QD:

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$$\Sigma^N = i\Gamma_N \mathbb{1}$$

$$\Sigma^S \approx -\frac{\Gamma_S}{\sqrt{\Delta^2 - \omega^2}} \begin{pmatrix} \omega & \Delta\delta_n \\ \Delta\delta_n & \omega \end{pmatrix}$$

Symmetric case
$$\begin{cases} \vec{r}_0 = (0,0) \\ t(\theta) = \text{cte.} \end{cases}$$

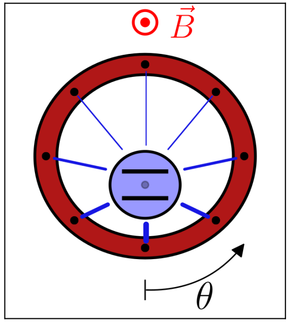
We write the Green's function for the QD:

$$G_{\sigma}^{QD}(\omega,\phi)=\left(\left(G_{0,\sigma}^{QD}(\omega,\phi)\right)^{-1}-\Sigma_{\sigma}^{S}(\omega,\phi)-\Sigma_{\sigma}^{N}-\Sigma_{\sigma}^{HFB}\right)^{-1}$$

$$\Sigma^{\mathrm{HFB}}=U\begin{pmatrix}\langle n_{-\sigma}\rangle & \langle d_{\sigma}d_{-\sigma}\rangle\\ \left\langle d_{\sigma}^{\dagger}d_{-\sigma}^{\dagger}\right\rangle & -\langle n_{\sigma}\rangle\end{pmatrix}$$

$$\Sigma^{N}=i\Gamma_{N}\mathbb{1}$$

$$\Sigma^{S}\approx-\frac{\Gamma_{S}}{\sqrt{\Delta^{2}-\omega^{2}}}\begin{pmatrix}\omega & \Delta\delta_{n}\\ \Delta\delta_{n} & \omega\end{pmatrix}$$



$$\Sigma^{\mathrm{S}}=\int d\theta d\bar{\theta}t(\theta+\bar{\theta})g_{\mathrm{S}}(\theta+\bar{\theta},\theta-\bar{\theta})t(\theta-\bar{\theta})$$

$$g_{\mathrm{S}}\sim\begin{pmatrix}\omega & \Delta e^{in\theta}\\ \Delta e^{-in\theta} & \omega\end{pmatrix}$$

Pairing term is zero for n>0 when the integral is done!
(in the symmetric coupling)

We write the Green's function for the QD:

$$G_{\sigma}^{QD}(\omega,\phi) = \left(\left(G_{0,\sigma}^{QD}(\omega,\phi) \right)^{-1} - \Sigma_{\sigma}^{S}(\omega,\phi) - \Sigma_{\sigma}^{N} - \Sigma_{\sigma}^{HFB} \right)^{-1}$$

$$\Sigma^{\text{HFB}} = U \begin{pmatrix} \langle n_{-\sigma} \rangle & \langle d_{\sigma} d_{-\sigma} \rangle \\ \left\langle d_{\sigma}^{\dagger} d_{-\sigma}^{\dagger} \right\rangle & -\langle n_{\sigma} \rangle \end{pmatrix}$$

$$\Sigma^{N} = i\Gamma_{N}\mathbb{1}$$

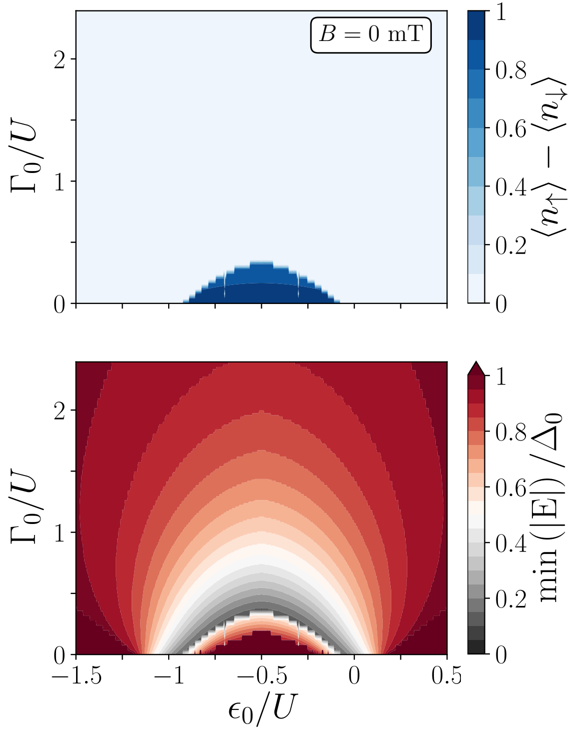
Σ^{S} = (Large expression)

We compute the LDOS in the QD and the occupation for different cases

$$\text{LDOS}(\omega,\phi) = -\frac{1}{\pi} \sum_{\sigma} \lim_{\eta \to 0} \text{Im}\left\{ \text{Tr}\left\{ \text{G}_{\sigma}^{\text{QD}}(\omega - \text{i}\eta, \phi) \right\} \right\}$$

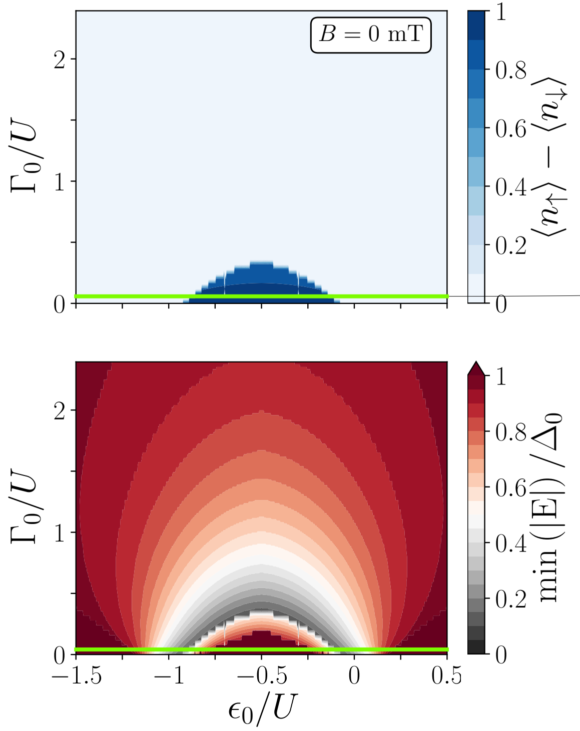
Results

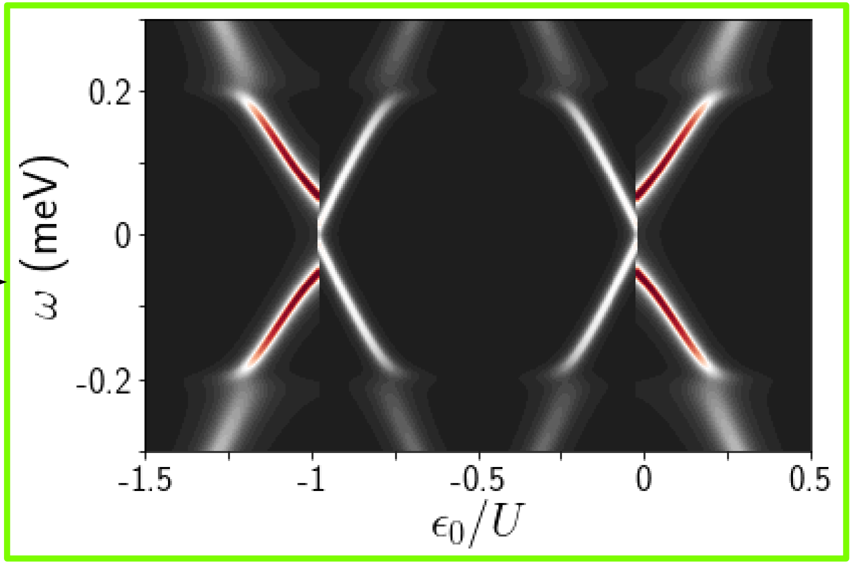
n=0
Symmetric case



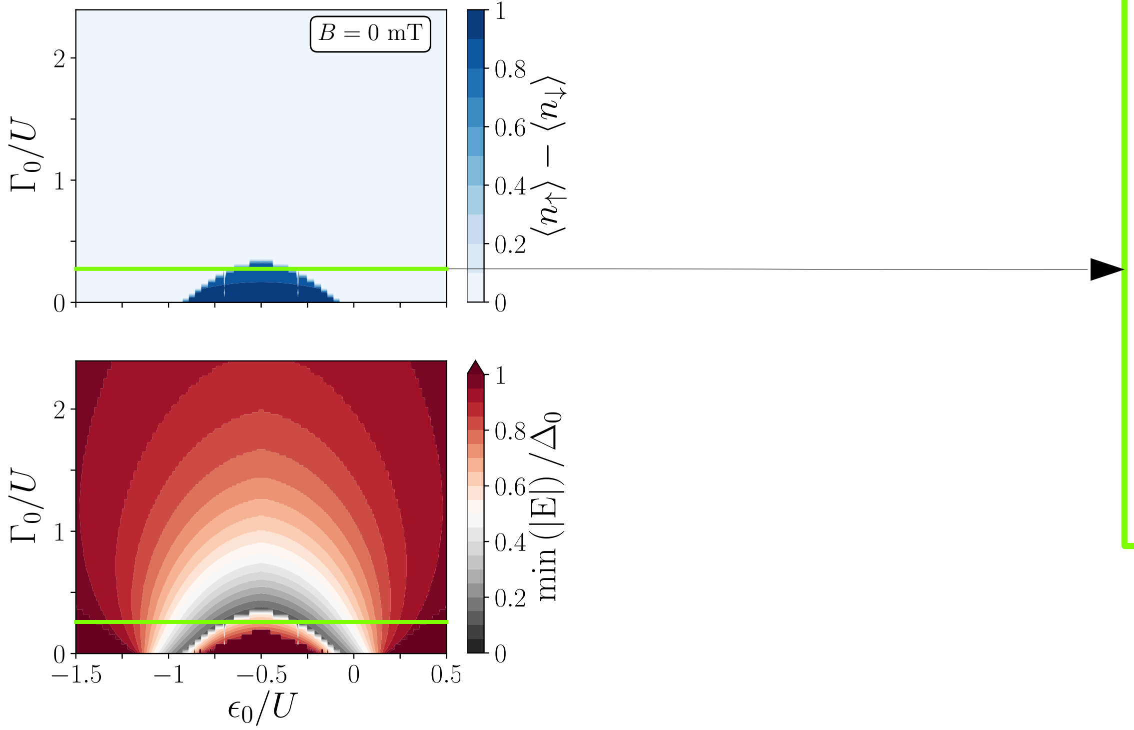
Results

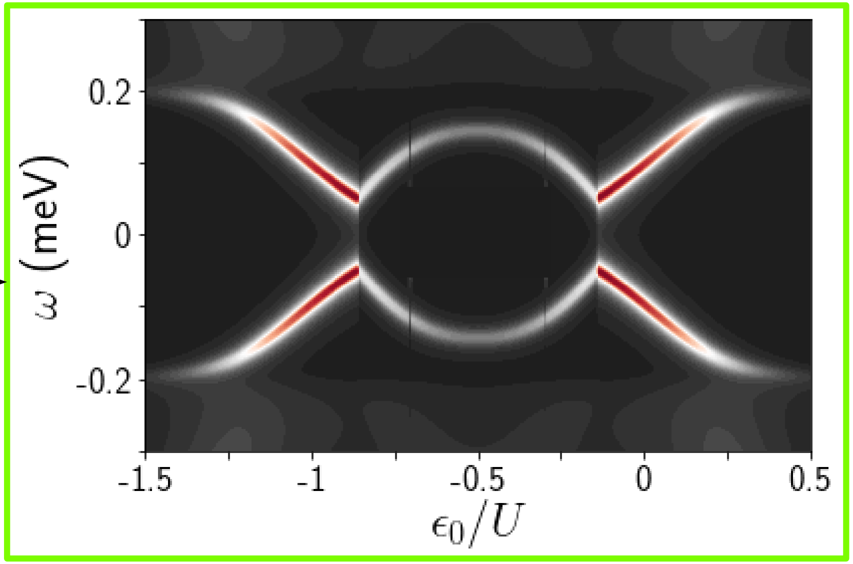
n=0
Symmetric case





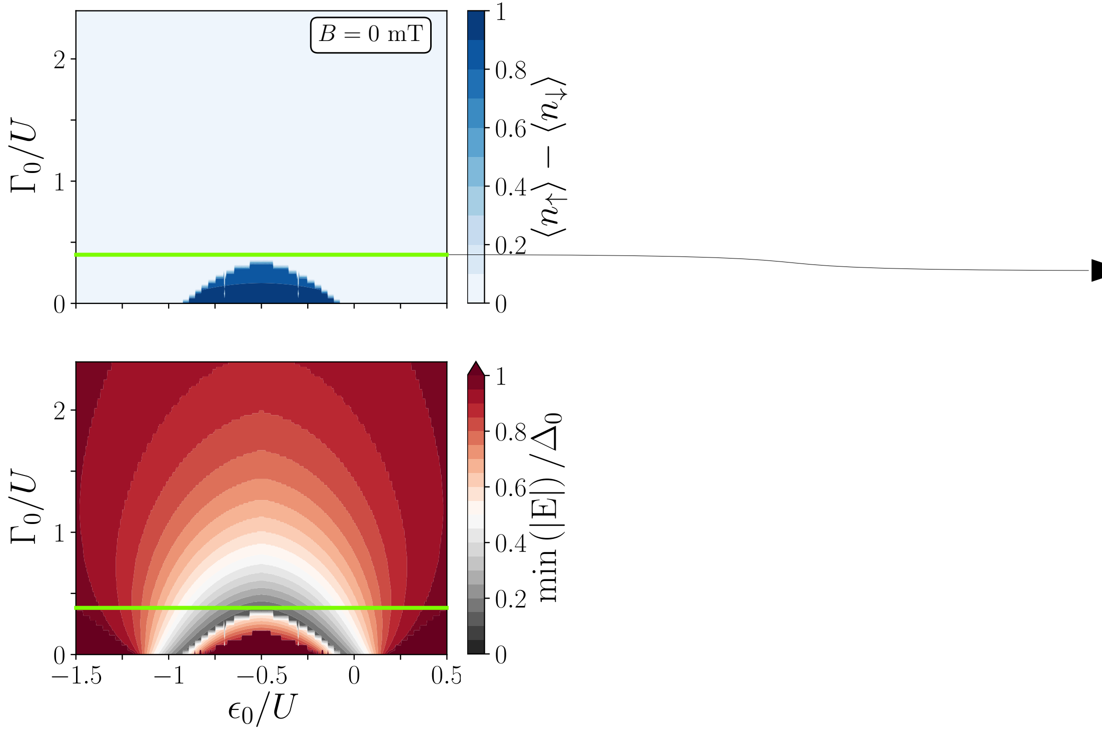
n=0
Symmetric case

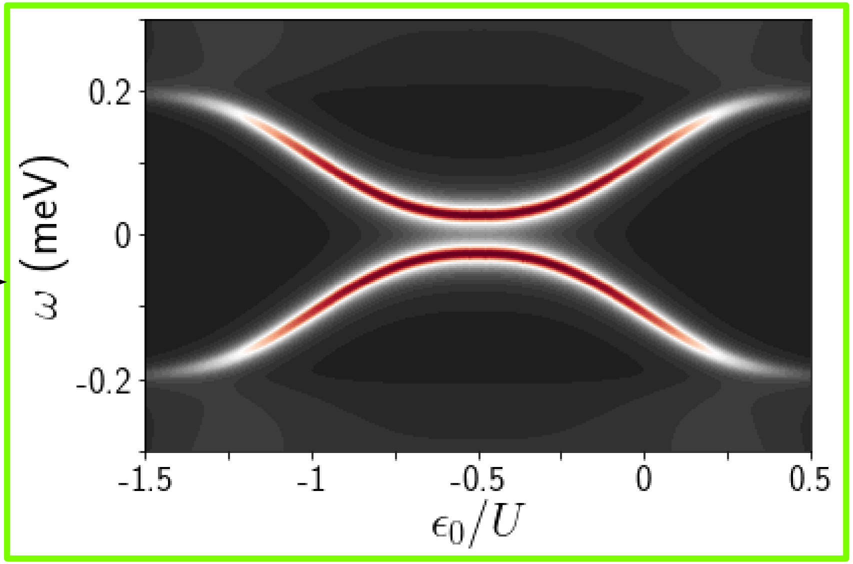




Results

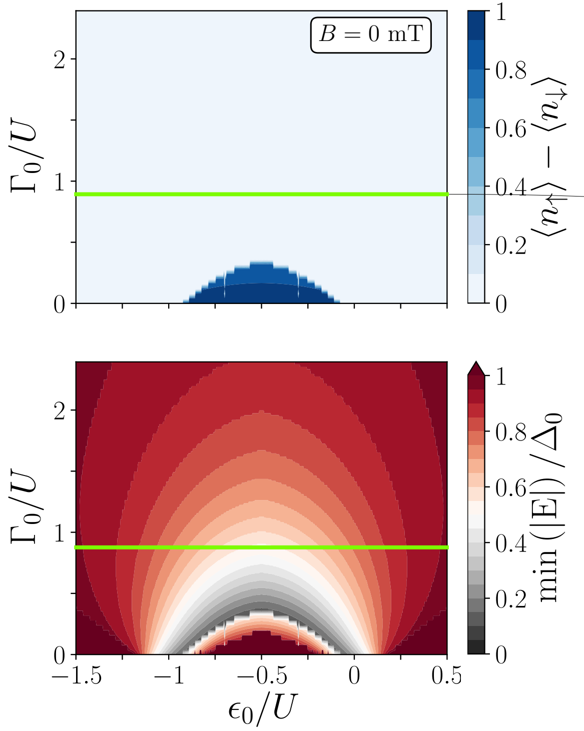
n=0
Symmetric case

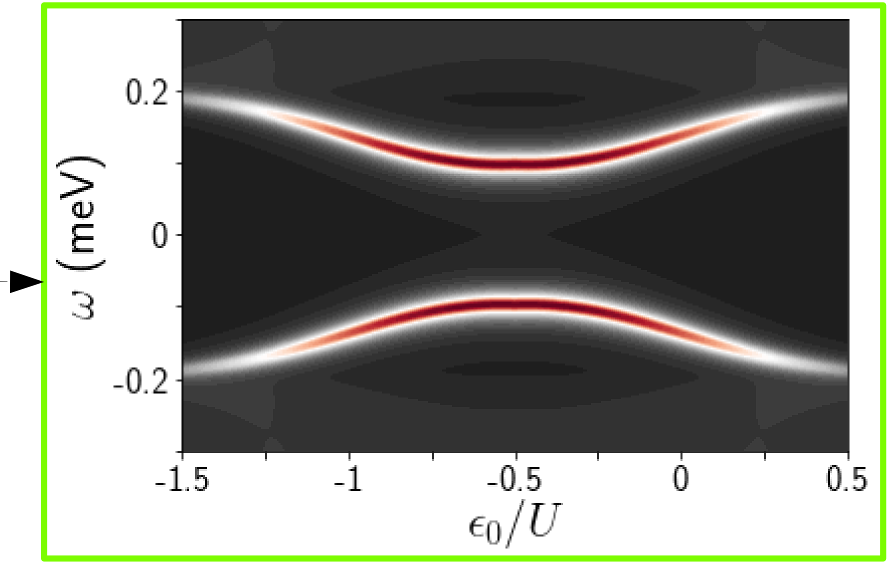




Results

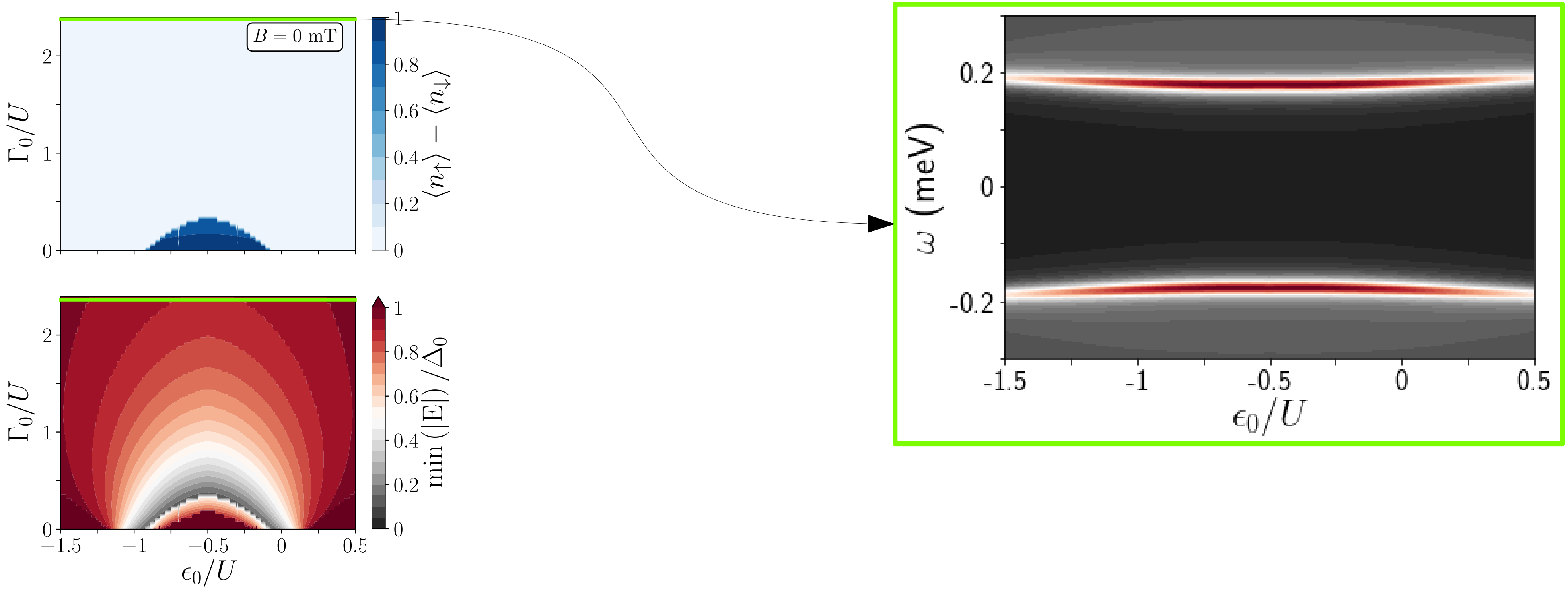
n=0
Symmetric case





Results

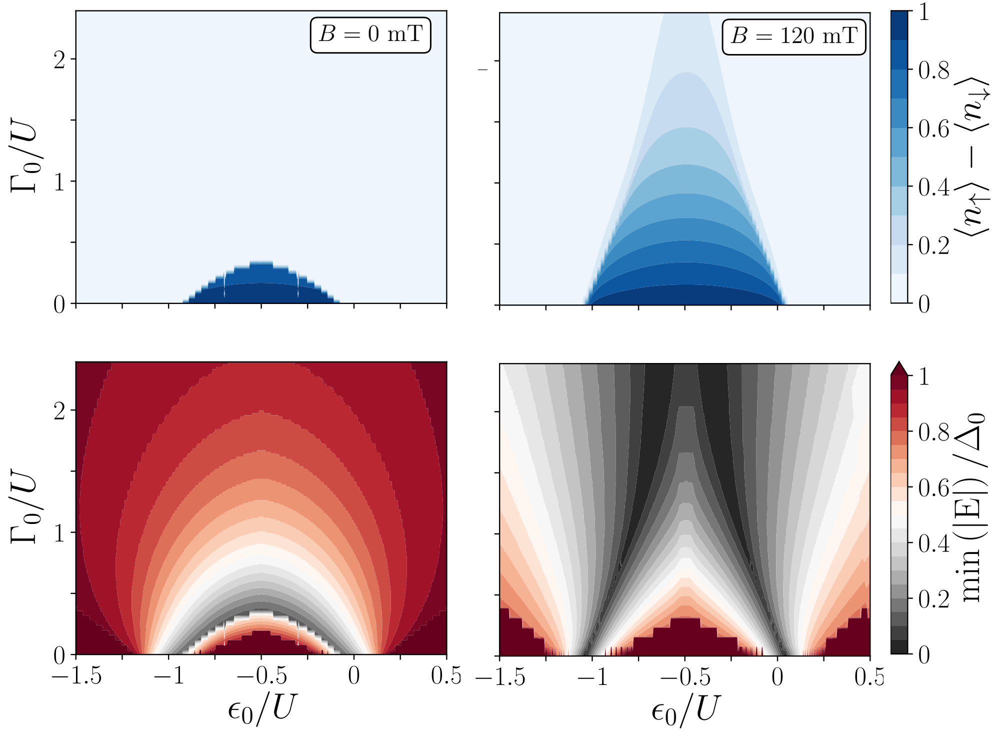
n=0
Symmetric case



Results

n=0
Symmetric case

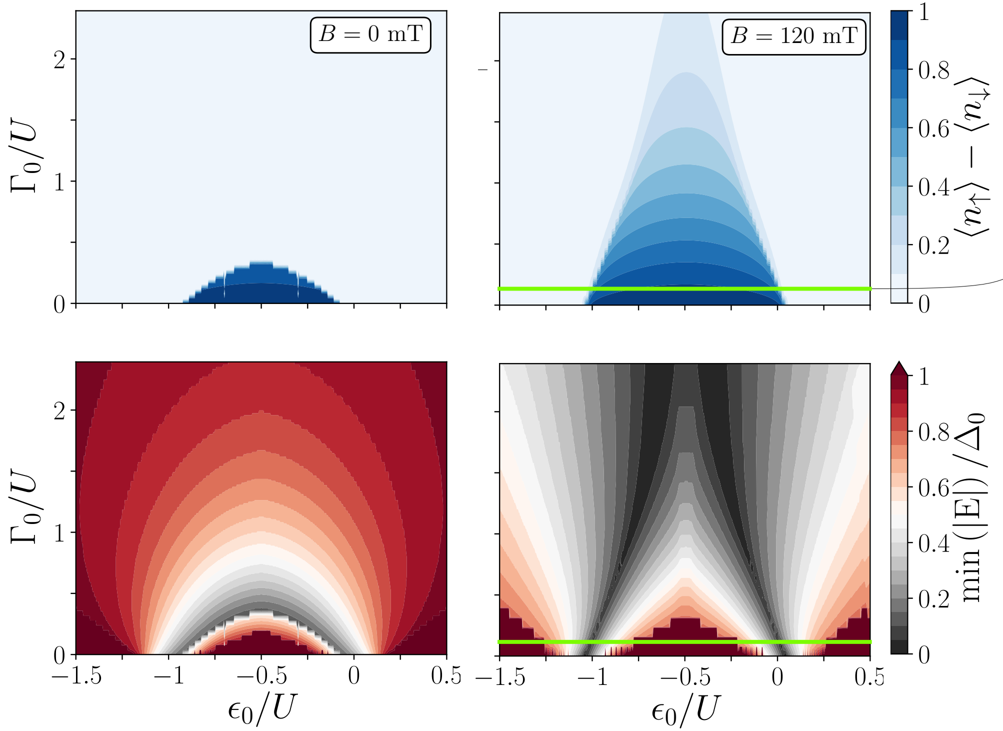
n=1
Symmetric case

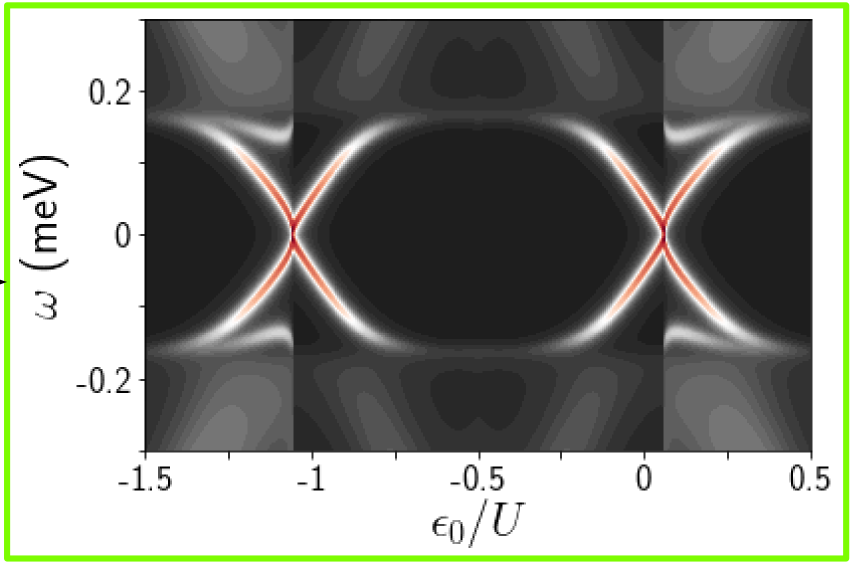


Results

n=0
Symmetric case

n=1
Symmetric case

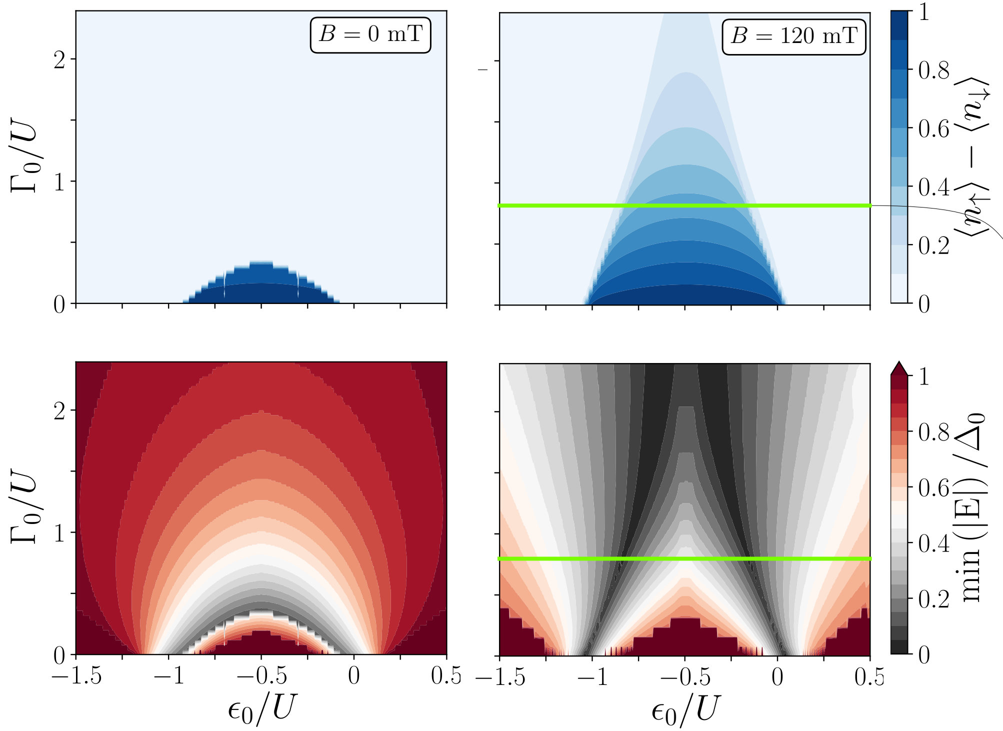


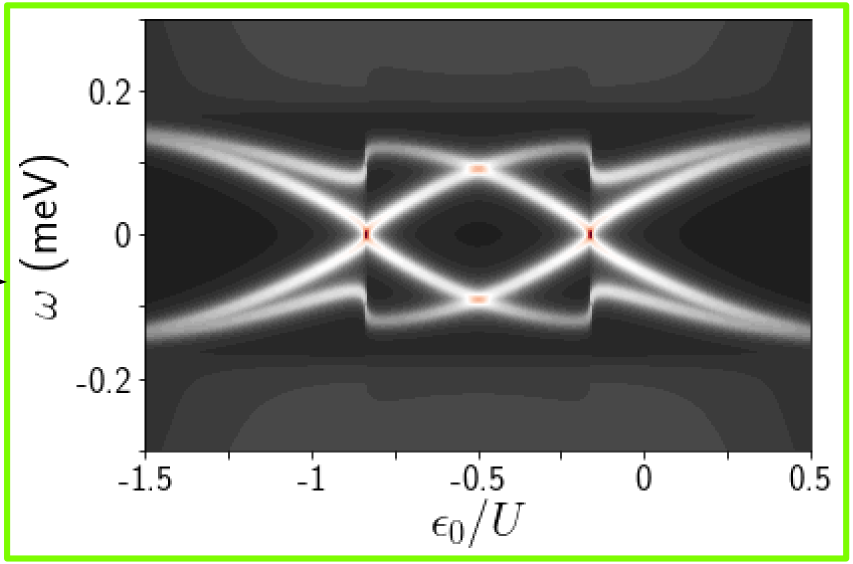


Results

n=0
Symmetric case

n=1
Symmetric case

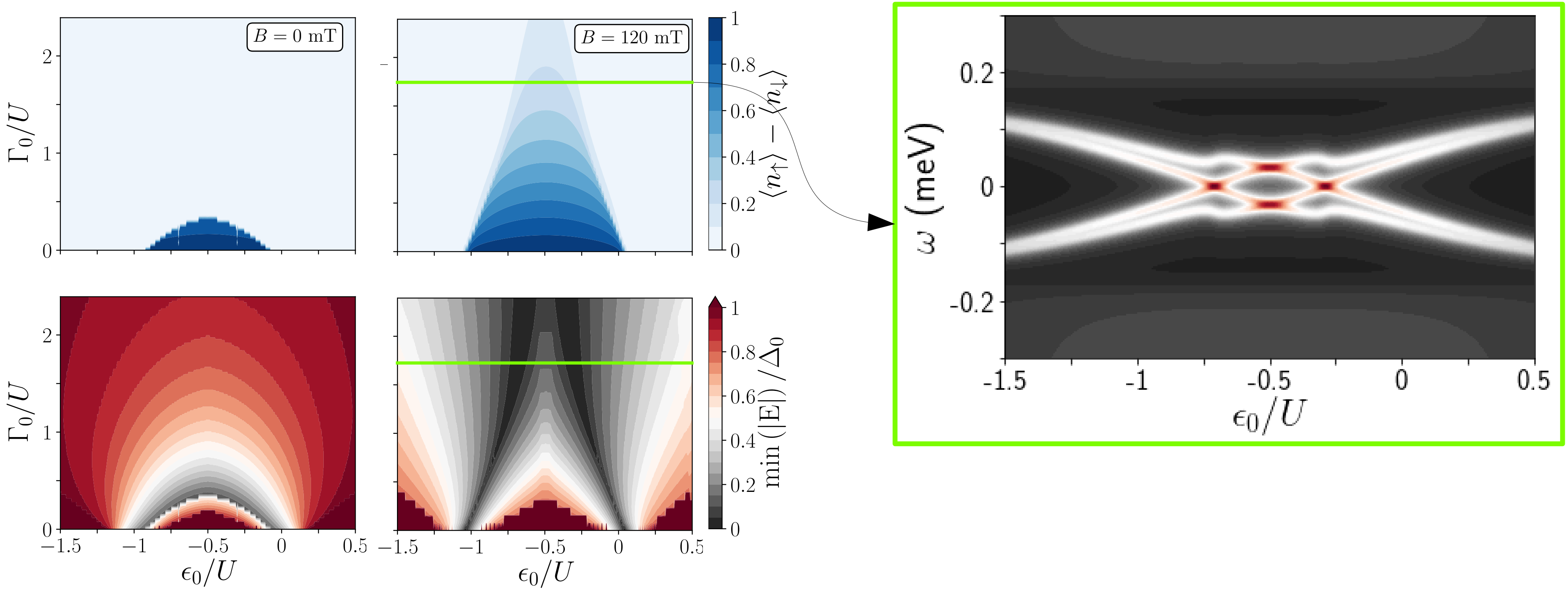




Results

n=0
Symmetric case

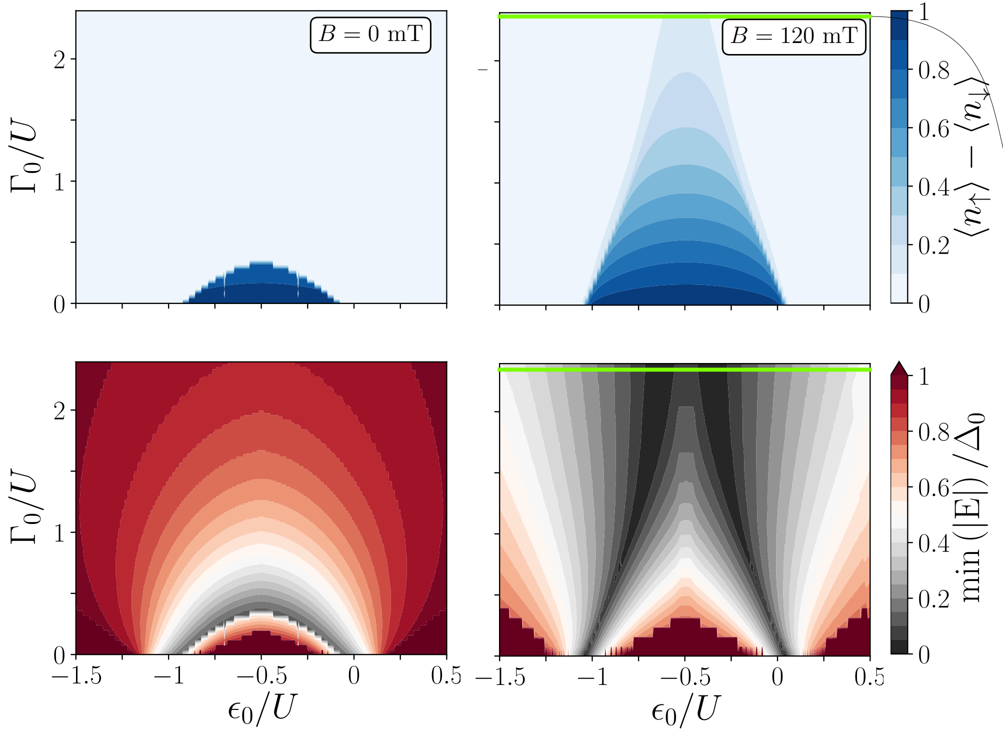
n=1
Symmetric case

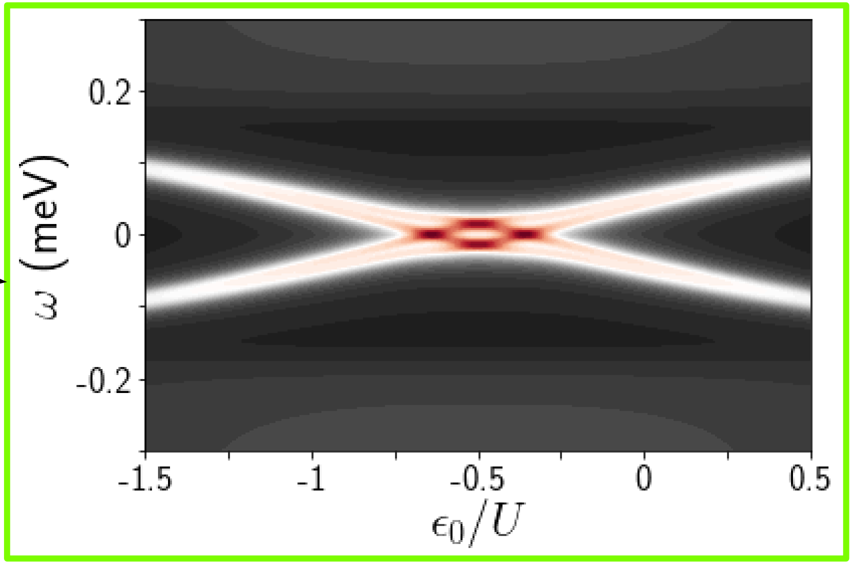


Results

n=0
Symmetric case

n=1
Symmetric case

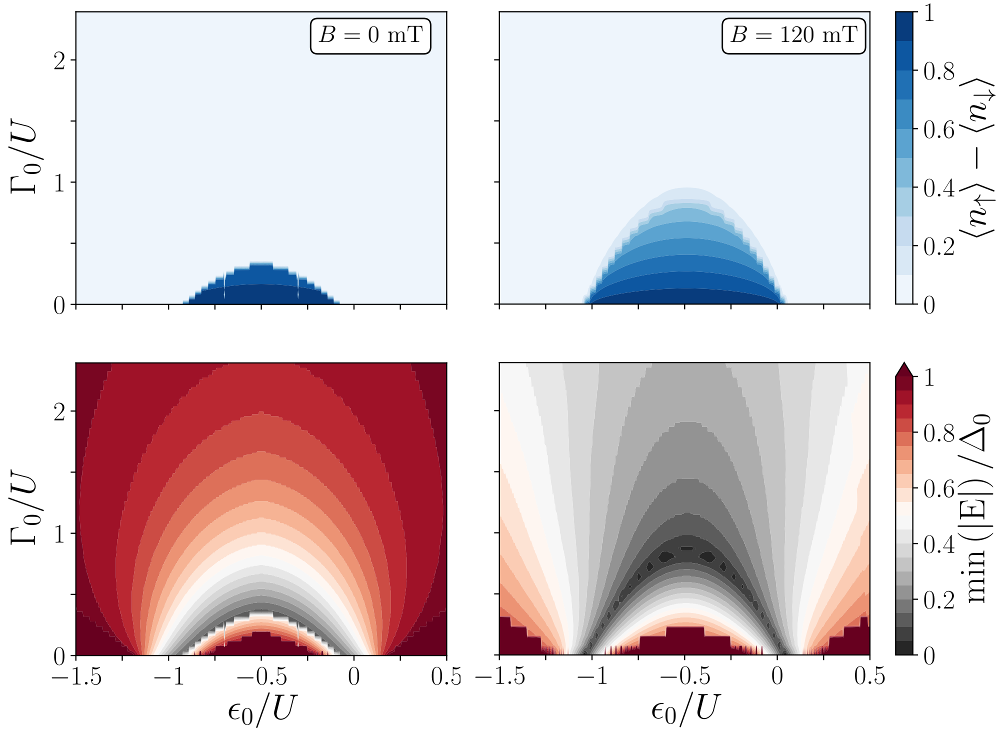




Results

n=0 Asymmetric case

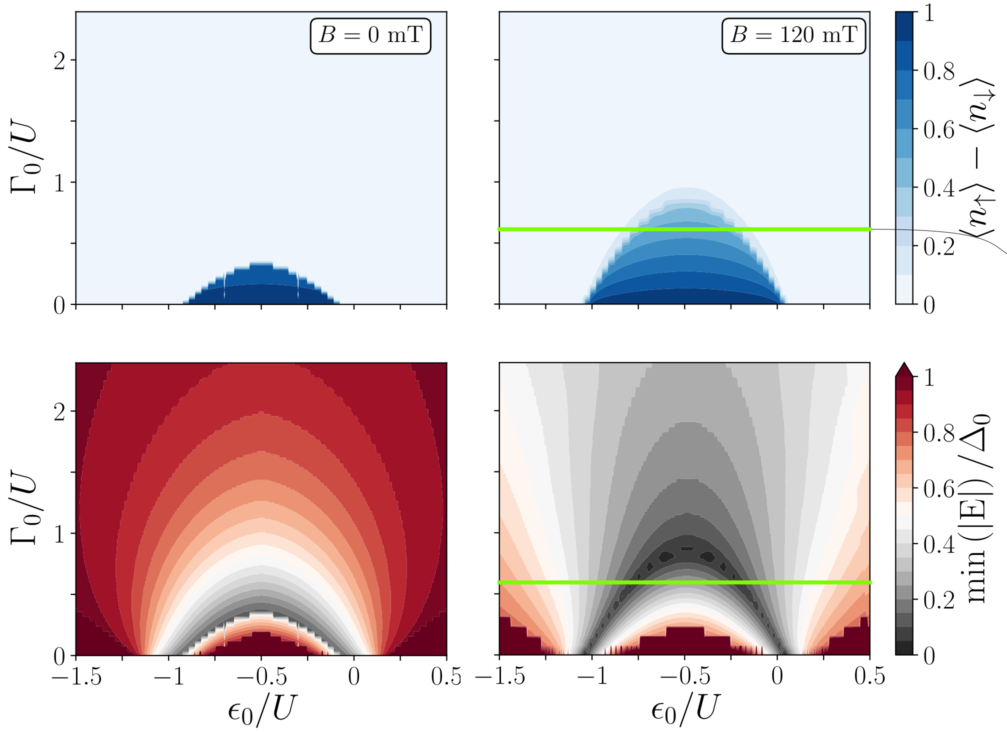
n=1 Asymmetric case

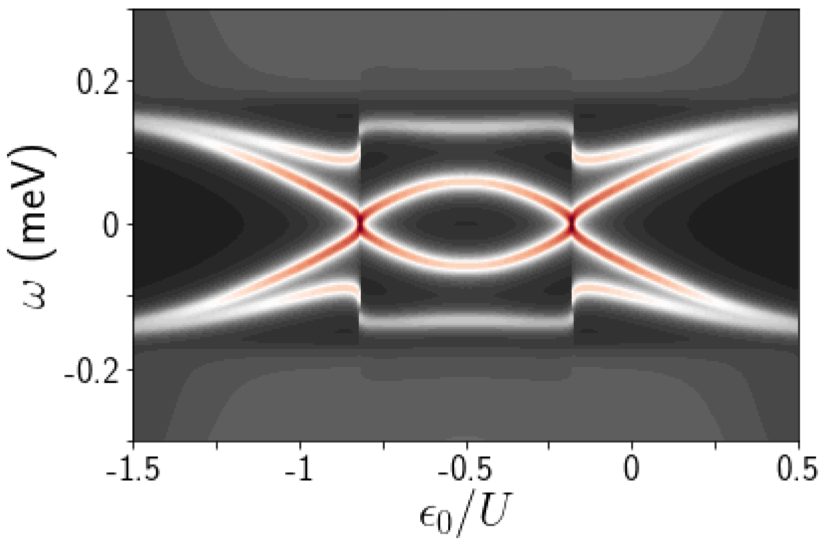


Results

n=0
Asymmetric case

n=1
Asymmetric case

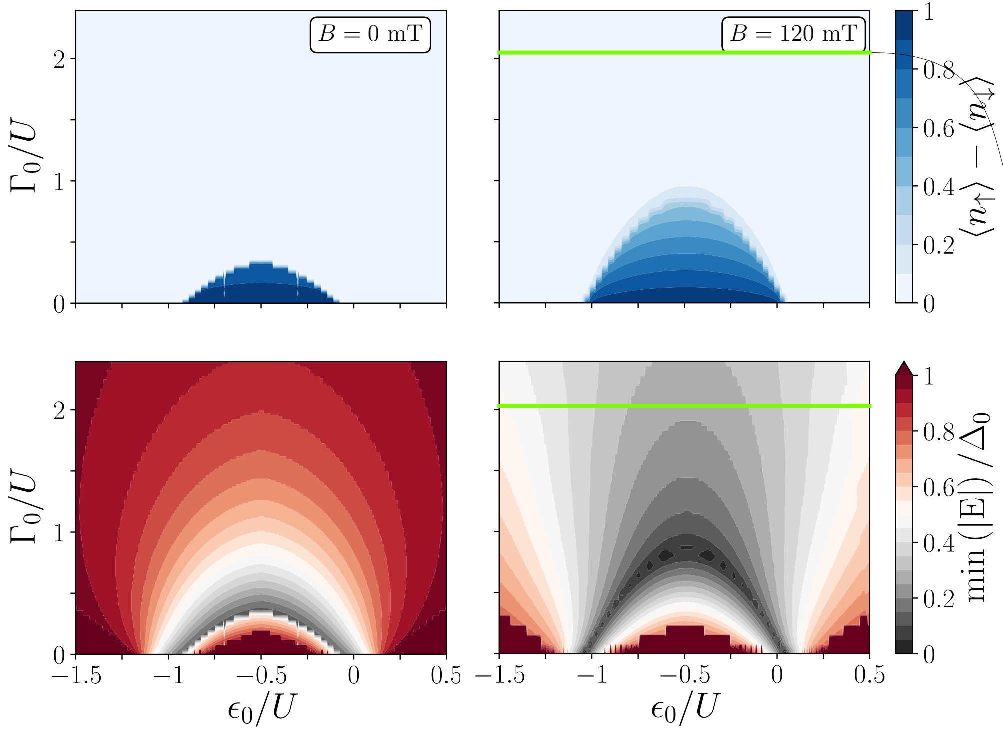


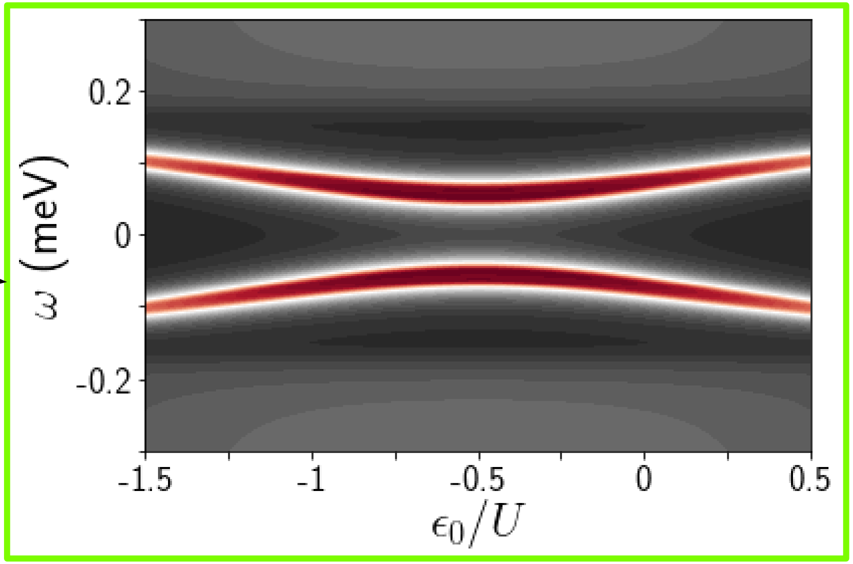


Results

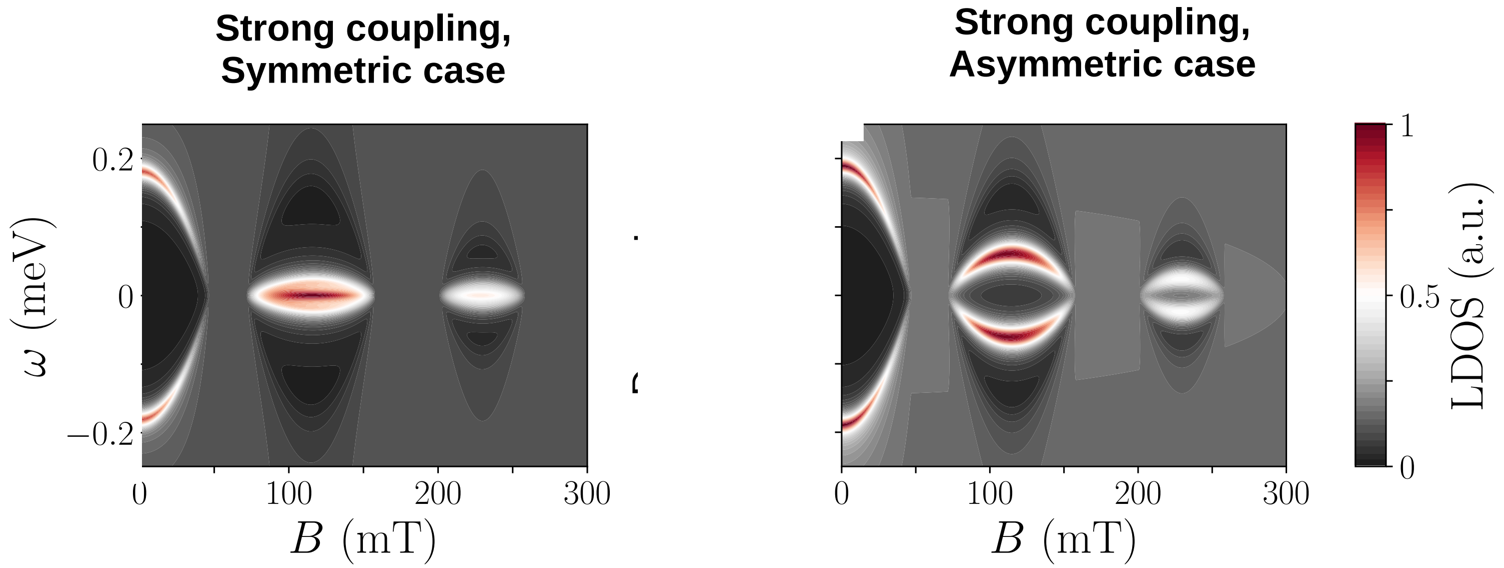
n=0
Asymmetric case

n=1
Asymmetric case



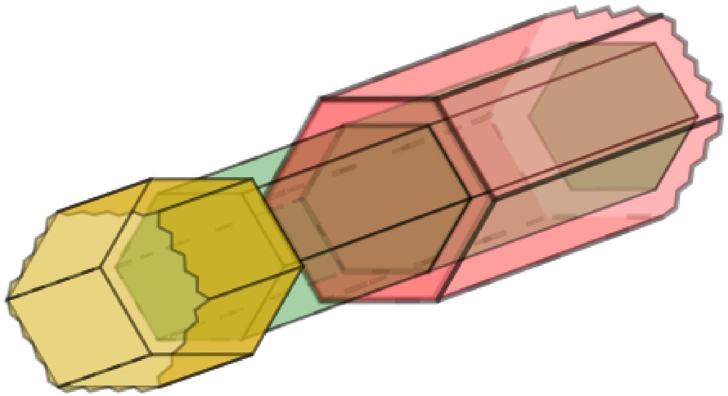


Results

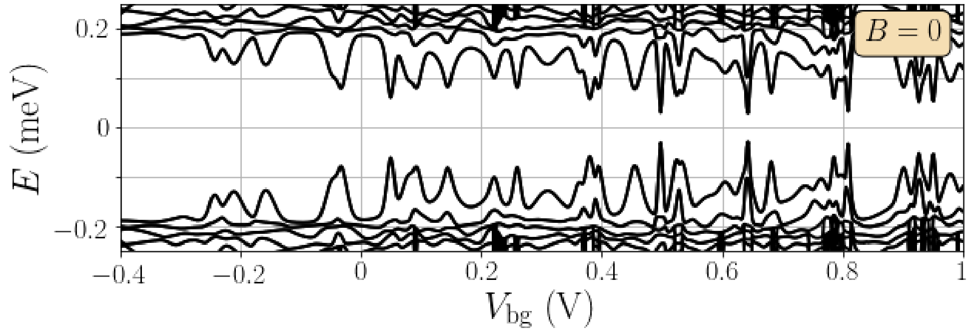


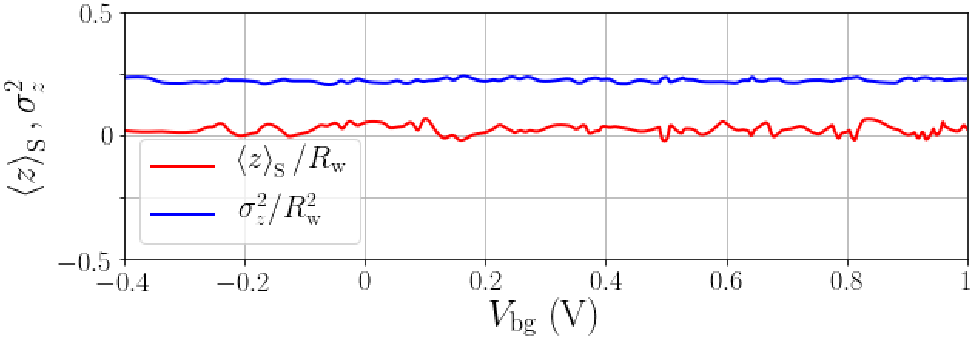
QD states may give rise to similar features to those of Majorana Bound States in full-shell wires

Results

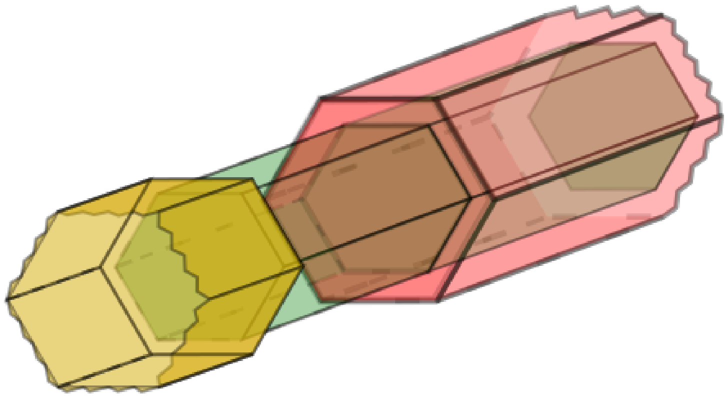


$$H = \left(\frac{\hbar^2 \vec{k}^2}{2m^*} + e\phi(\vec{r})\right) \sigma_0 \tau_z + \Delta(\vec{r}) \sigma_y \tau_y$$
$$+ \frac{1}{2}\left[\vec{\alpha}(\vec{r}) \cdot (\vec{\sigma} \times \vec{k}) + (\vec{\sigma} \times \vec{k}) \cdot \vec{\alpha}(\vec{r})\right] \tau_z$$

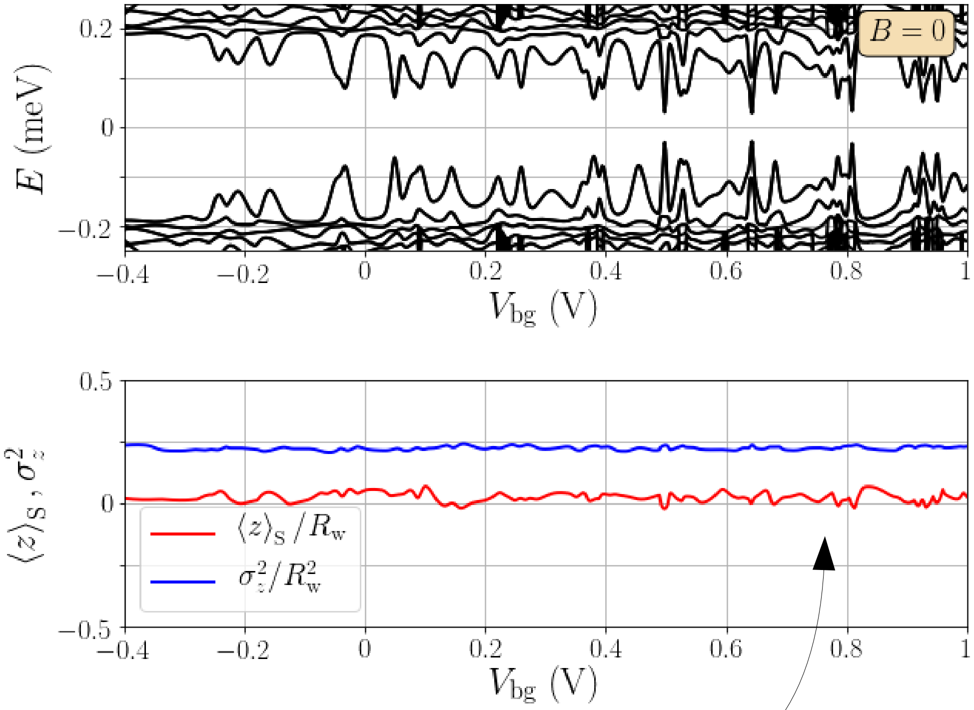




Results



$$H = \left(\frac{\hbar^2 \vec{k}^2}{2m^*} + e\phi(\vec{r})\right)\sigma_0\tau_z + \Delta(\vec{r})\sigma_y\tau_y$$
$$+\frac{1}{2}\left[\vec{\alpha}(\vec{r})\cdot(\vec{\sigma}\times\vec{k}) + (\vec{\sigma}\times\vec{k})\cdot\vec{\alpha}(\vec{r})\right]\tau_z$$

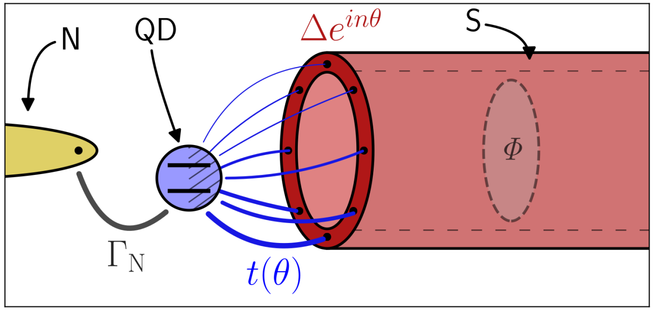


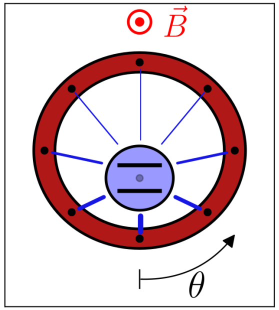
Symmetric case is a realistic situation!

Conclusions

Take-home messages

- N-QD-S junctions with full-shell superconductors develop quite different features and phase diagrams, specially in some cases.
- In the symmetric case, the superconducting pairing is always suppressed for the n>0 lobes.
- The YSR states are further pushed towards zero energy compared to a conventional N-QD-S junction.





Supplementary Material

A: Equations

Equations

$$\left(\Sigma_\sigma^S(\omega;n)\right)_{00} = -\rho_S\pi\left(2\pi t_0\right)^2 \frac{\omega+\frac{n}{2}t_\theta\left(n-\frac{\phi}{\phi_0}\right)}{\sqrt{\Delta^2-\left(\omega+\frac{n}{2}t_\theta\left(n-\frac{\phi}{\phi_0}\right)\right)^2}}$$

$$-\rho_S\pi\sum_{k=1}^{\infty}\left(\pi t_k\right)^2\left\{\frac{\omega+\left(\frac{n}{2}+k\right)t_\theta\left(n-\frac{\phi}{\phi_0}\right)}{\sqrt{\Delta^2-\left(\omega+\left(\frac{n}{2}+k\right)t_\theta\left(n-\frac{\phi}{\phi_0}\right)\right)^2}}+\frac{\omega+\left(\frac{n}{2}-k\right)t_\theta\left(n-\frac{\phi}{\phi_0}\right)}{\sqrt{\Delta^2-\left(\omega+\left(\frac{n}{2}-k\right)t_\theta\left(n-\frac{\phi}{\phi_0}\right)\right)^2}}\right\}, \quad (96)$$

$$\left(\Sigma_\sigma^S(\omega;n)\right)_{11} = -\rho_S\pi\left(2\pi t_0\right)^2 \frac{\omega-\frac{n}{2}t_\theta\left(n-\frac{\phi}{\phi_0}\right)}{\sqrt{\Delta^2-\left(\omega-\frac{n}{2}t_\theta\left(n-\frac{\phi}{\phi_0}\right)\right)^2}}$$

$$-\rho_S\pi\sum_{k=1}^{\infty}\left(\pi t_k\right)^2\left\{\frac{\omega-\left(\frac{n}{2}-k\right)t_\theta\left(n-\frac{\phi}{\phi_0}\right)}{\sqrt{\Delta^2-\left(\omega-\left(\frac{n}{2}-k\right)t_\theta\left(n-\frac{\phi}{\phi_0}\right)\right)^2}}+\frac{\omega-\left(\frac{n}{2}+k\right)t_\theta\left(n-\frac{\phi}{\phi_0}\right)}{\sqrt{\Delta^2-\left(\omega-\left(\frac{n}{2}+k\right)t_\theta\left(n-\frac{\phi}{\phi_0}\right)\right)^2}}\right\}, \quad (97)$$

Equations

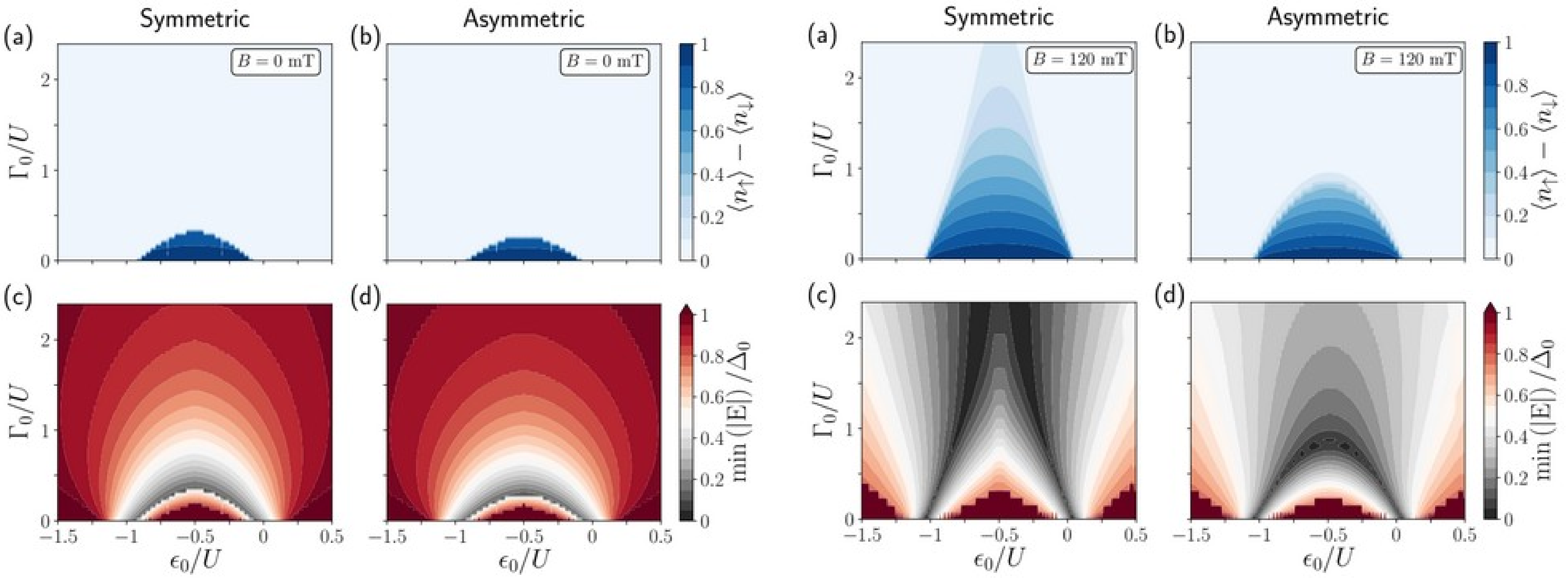
$$
\begin{aligned}
\left(\Sigma_\sigma^S(\omega;n)\right)_{01} = -\rho_S \pi \delta_{n,0} \Bigg\{ & (2\pi t_0) \frac{\Delta}{\sqrt{\Delta^2 - \left(\omega + \frac{n}{2} t_\theta \left(n - \frac{\phi}{\phi_0}\right)\right)^2}} \\
& + \sum_{k=1}^{\infty} (\pi t_k)^2 \left[\frac{\Delta}{\sqrt{\Delta^2 - \left(\omega + \left(\frac{n}{2} + k\right) t_\theta \left(n - \frac{\phi}{\phi_0}\right)\right)^2}} + \frac{\Delta}{\sqrt{\Delta^2 - \left(\omega + \left(\frac{n}{2} - k\right) t_\theta \left(n - \frac{\phi}{\phi_0}\right)\right)^2}} \right] \Bigg\} \\
& - \rho_S \pi \left(1 - \delta_{n,0}\right) \Bigg\{ (2\pi t_0)^2 \frac{t_n}{2 t_0} \frac{\Delta}{\sqrt{\Delta^2 - \left(\omega + \frac{n}{2} t_\theta \left(n - \frac{\phi}{\phi_0}\right)\right)^2}} \\
& + \sum_{k=1}^{\infty} \left(1 - \delta_{k,n}\right) (2\pi t_0)^2 \frac{t_k t_{k-n}}{2 t_0^2} \frac{\Delta}{\sqrt{\Delta^2 - \left(\omega - \left(\frac{n}{2} - k\right) t_\theta \left(n - \frac{\phi}{\phi_0}\right)\right)^2}} \\
& + \left(1 - \delta_{k,-n}\right) (2\pi t_0)^2 \frac{t_k t_{k+n}}{2 t_0^2} \frac{\Delta}{\sqrt{\Delta^2 - \left(\omega - \left(\frac{n}{2} + k\right) t_\theta \left(n - \frac{\phi}{\phi_0}\right)\right)^2}} \Bigg\}. \qquad (99)
\end{aligned}
$$

Supplementary Material

B: Phase diagrams for different n

Phase diagrams

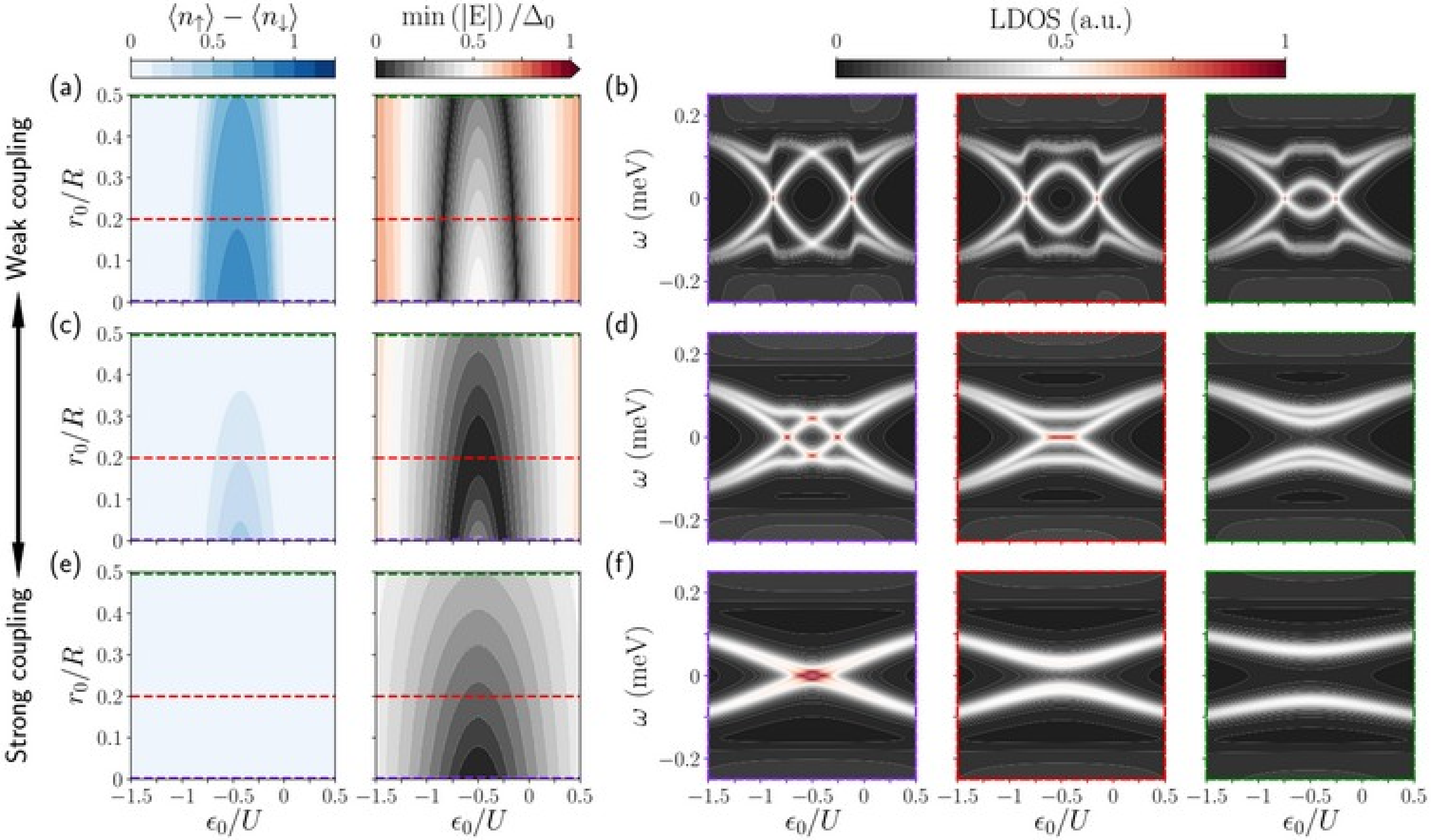
n=0 **n=1**



Supplementary Material

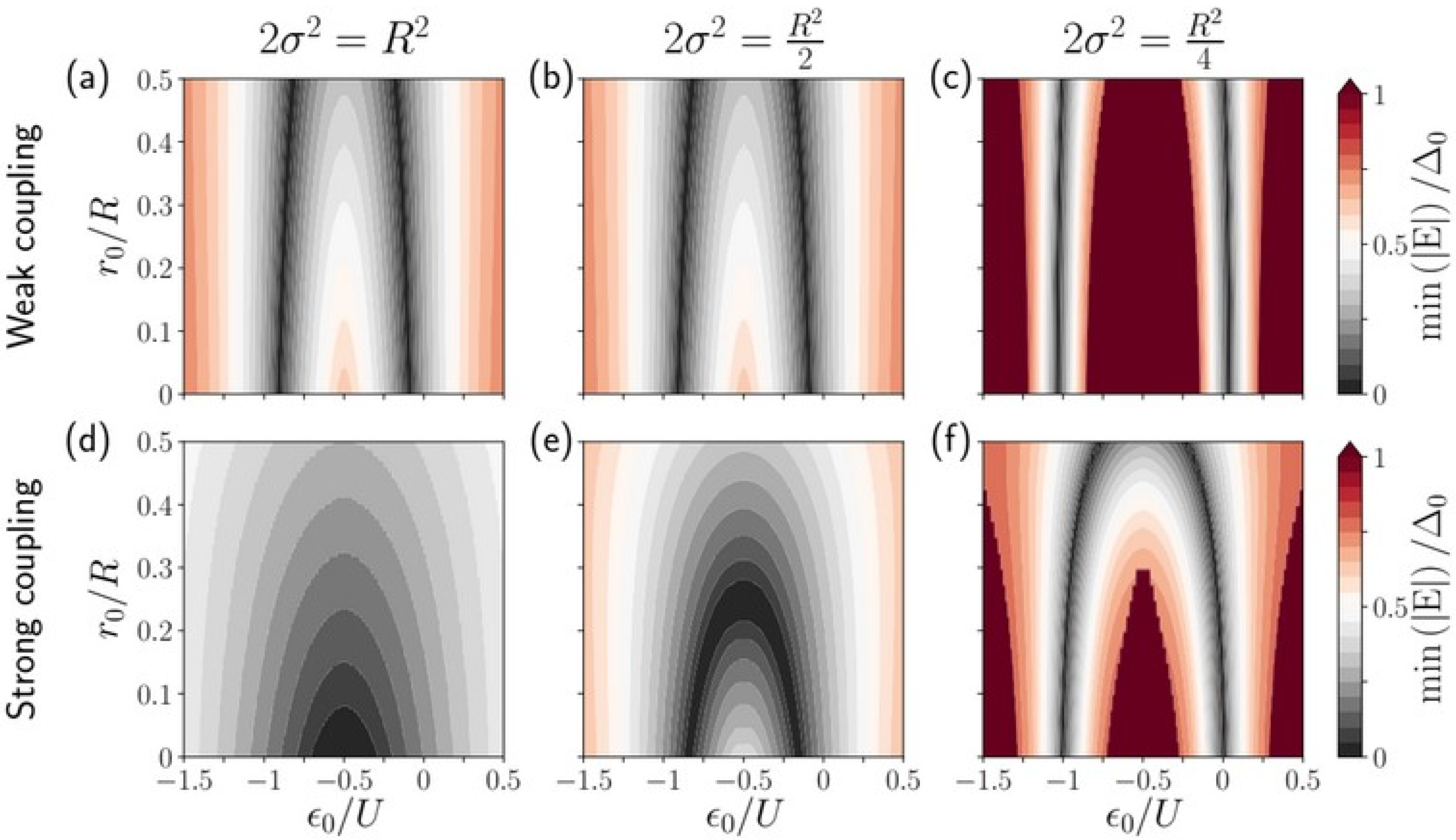
C: LDOS versus dot position and dot width

LDOS vs r_0



$\langle n_\uparrow \rangle - \langle n_\downarrow \rangle$
min (|E|) / Δ_0
LDOS (a.u.)
(a)
(b)
(c)
(d)
(e)
(f)
Weak coupling
Strong coupling
r_0/R
ω (meV)
ϵ_0/U

LDOS vs α

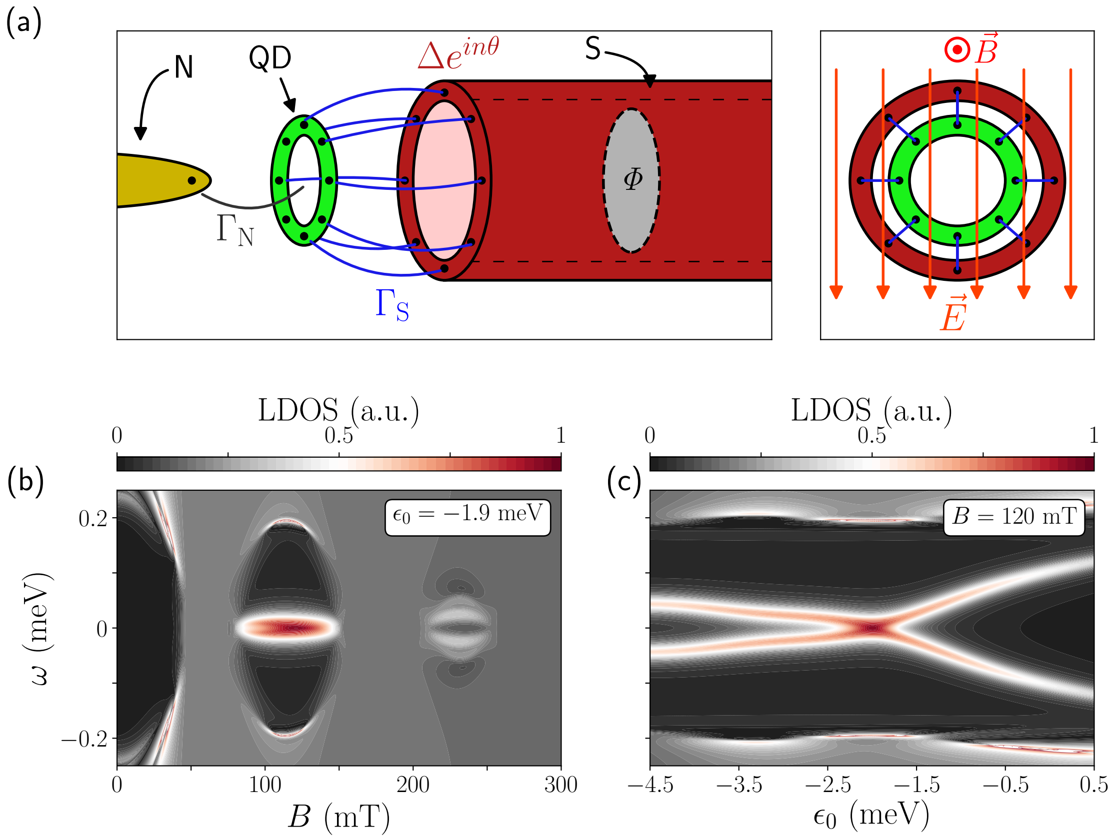


$2\sigma^2 = R^2$
$2\sigma^2 = \frac{R^2}{2}$
$2\sigma^2 = \frac{R^2}{4}$
(a)
(b)
(c)
(d)
(e)
(f)
Weak coupling
Strong coupling
r_0/R
ϵ_0/U
$\min(|E|)/\Delta_0$

Supplementary Material

D: Ring-shaped QD

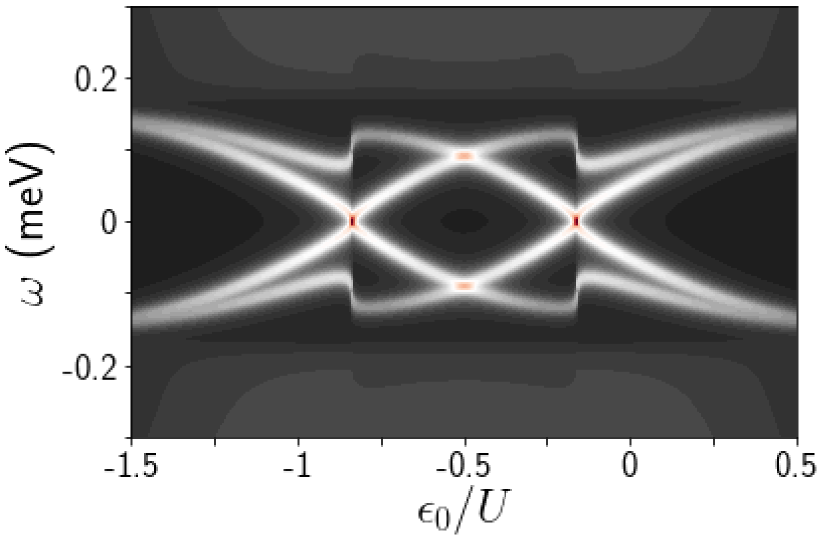
Ring-shpaed QD



Supplementary Material

E: Comparison to a S-QD-S junction

Comparison to SDS



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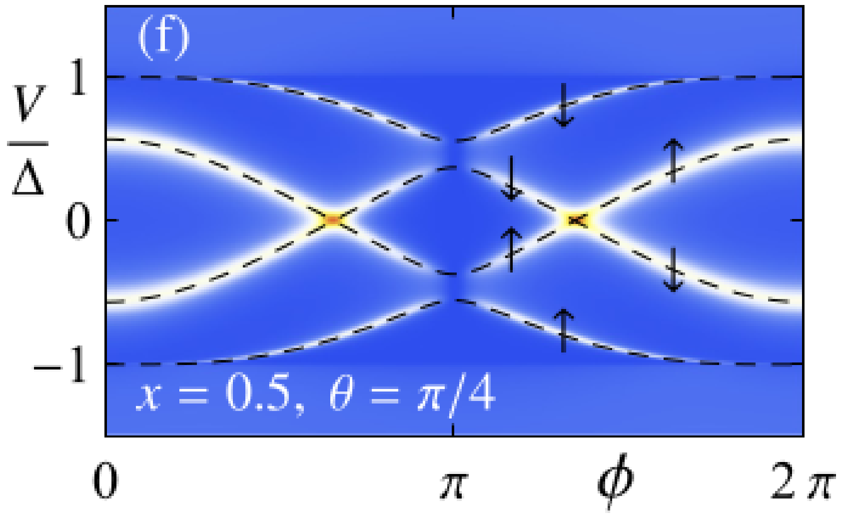
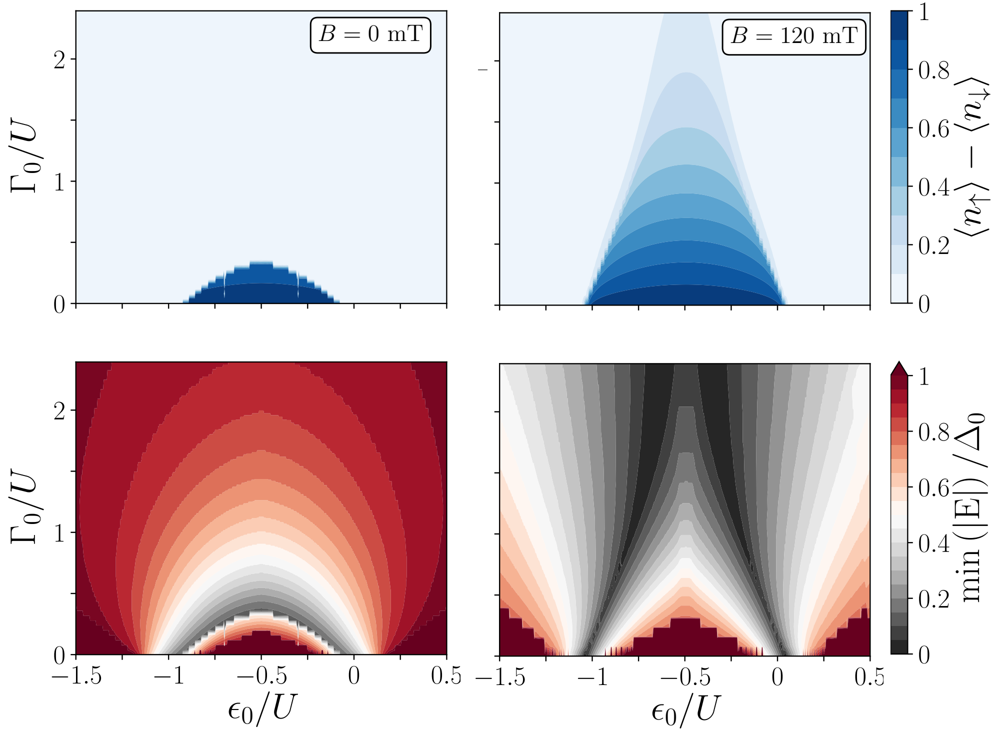


Comparison to SDS

n=0
Symmetric case

n=1
Symmetric case



M. Žonda et al. Scientific Reports 5, 8821 (2015)