Effect of the Electrostatic Environment in Majorana Nanowires

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Outline

1. Introduction
   a) Minimal theory of Majorana nanowires
   b) Experimental status and motivation

2. Results
   a) Model of the electrostatic environment
   b) Results

3. Conclusions

1. Introduction

- A Majorana particle is a fermion that is its own antiparticle. They correspond to solutions of the Dirac equation with $\gamma = \gamma^\dagger$.

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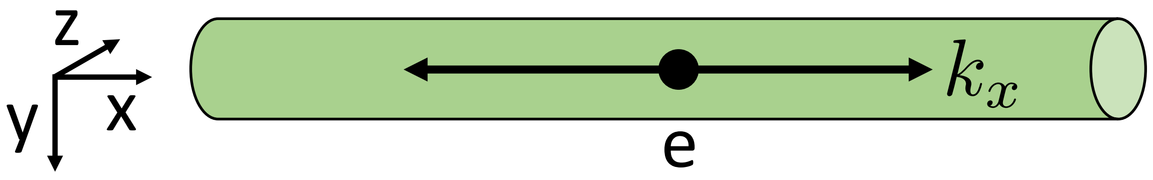
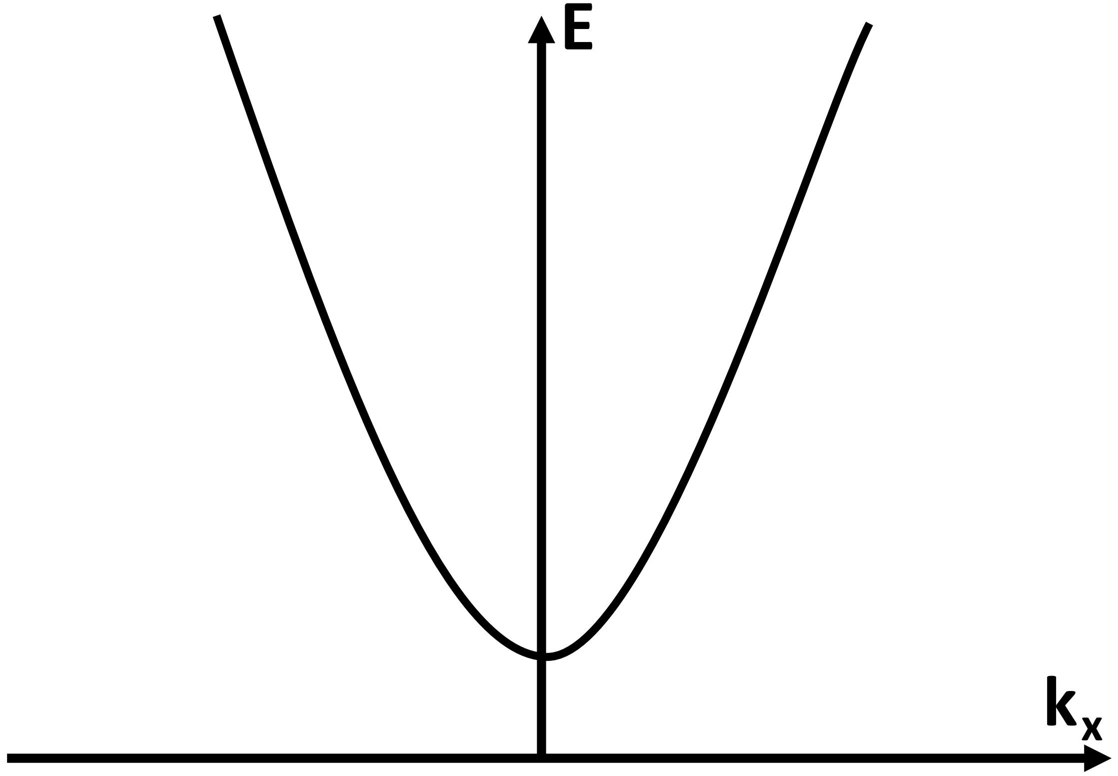
- A Majorana particle is a fermion that is its own antiparticle. They correspond to solutions of the Dirac equation with $\gamma = \gamma^\dagger$.
- Superconductivity violates charge conservation.
- In spinless p-wave superconductors, Bogoliubov quasi-particles can satisfy $\gamma = \gamma^\dagger$.
- Type p superconductivity is induced in semiconductor nanowires with:
  - proximity effect to type s superconductors
  - high spin-orbit coupling
  - And applying an external magnetic field.

1.a) Minimal theory

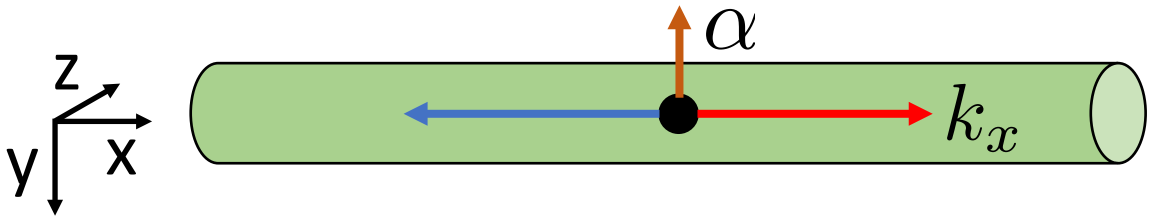


$$\hat{H}_0 = \left[\left(\hbar^2 k_x^2/2m - \mu\right)\sigma_0\right]\tau_z$$

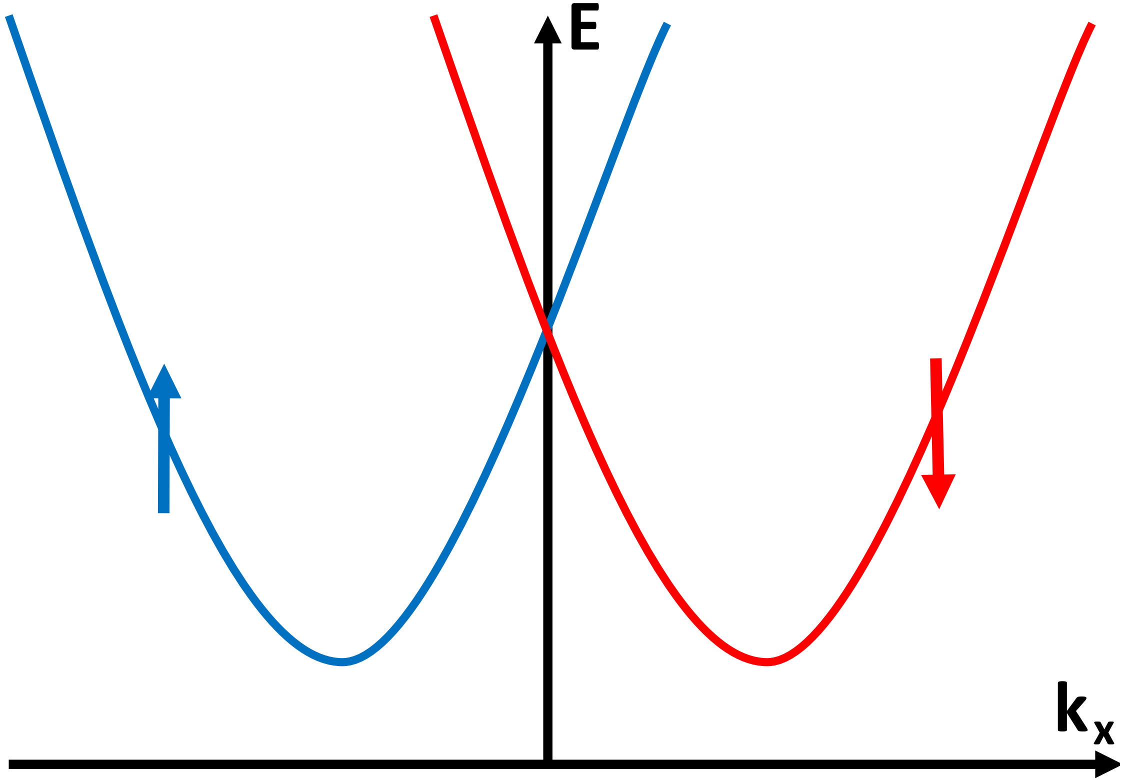
Dispersion relation



- High spin-orbit coupling (Rashba effect) α

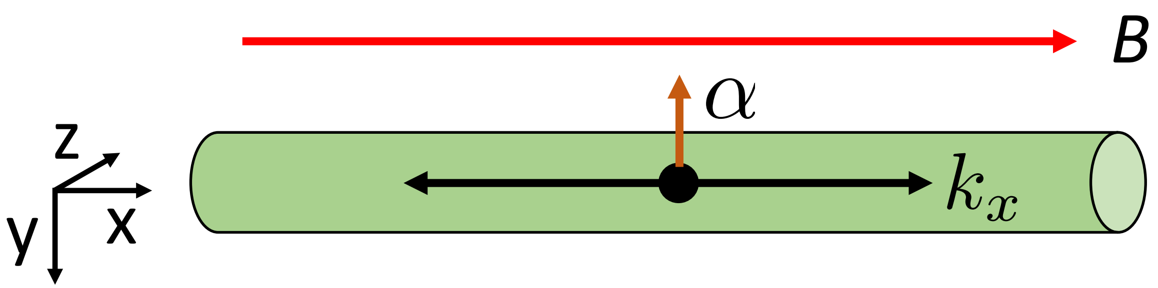


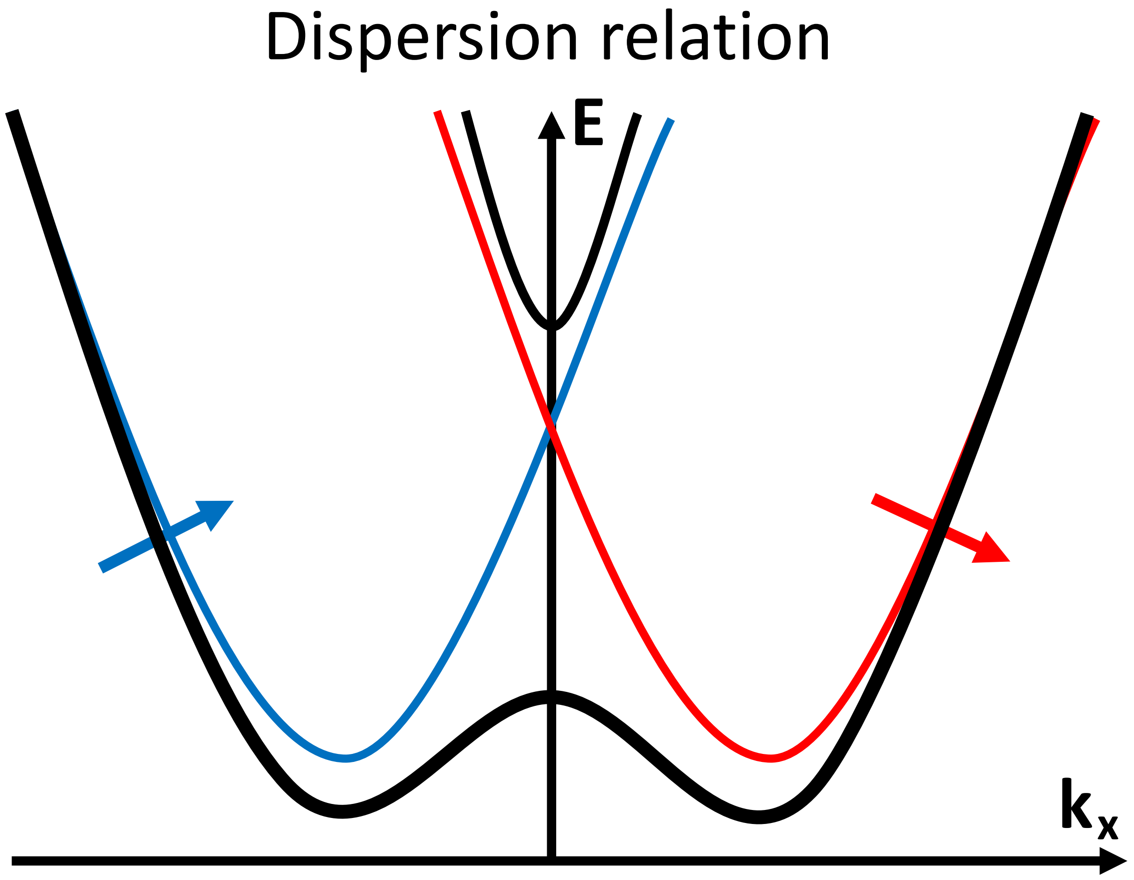
Dispersion relation



$$\hat{H}_0 = \left[\left(\hbar^2 k_x^2/2m - \mu\right)\sigma_0 + \alpha\sigma_y k_x\right]\tau_z$$

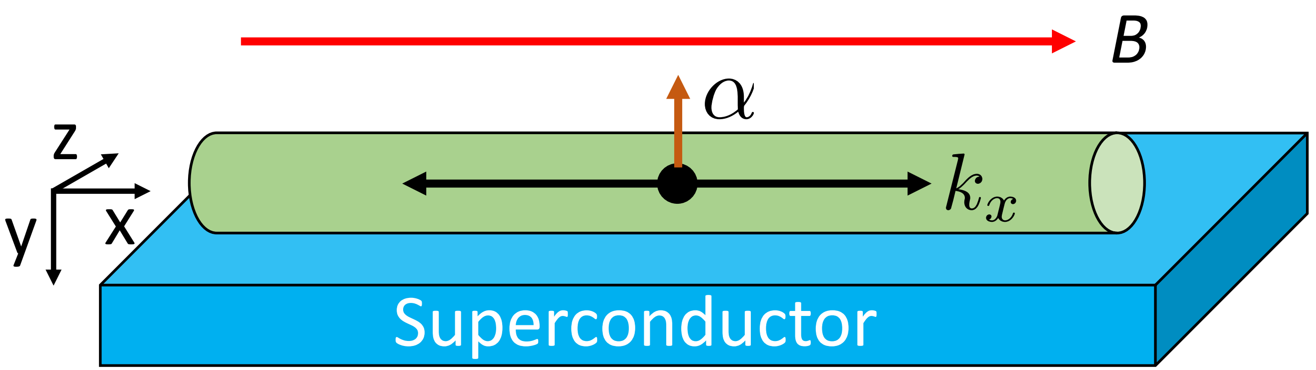
- High spin-orbit coupling (Rashba effect) α
- External magnetic field (Zeeman splitting) V_Z

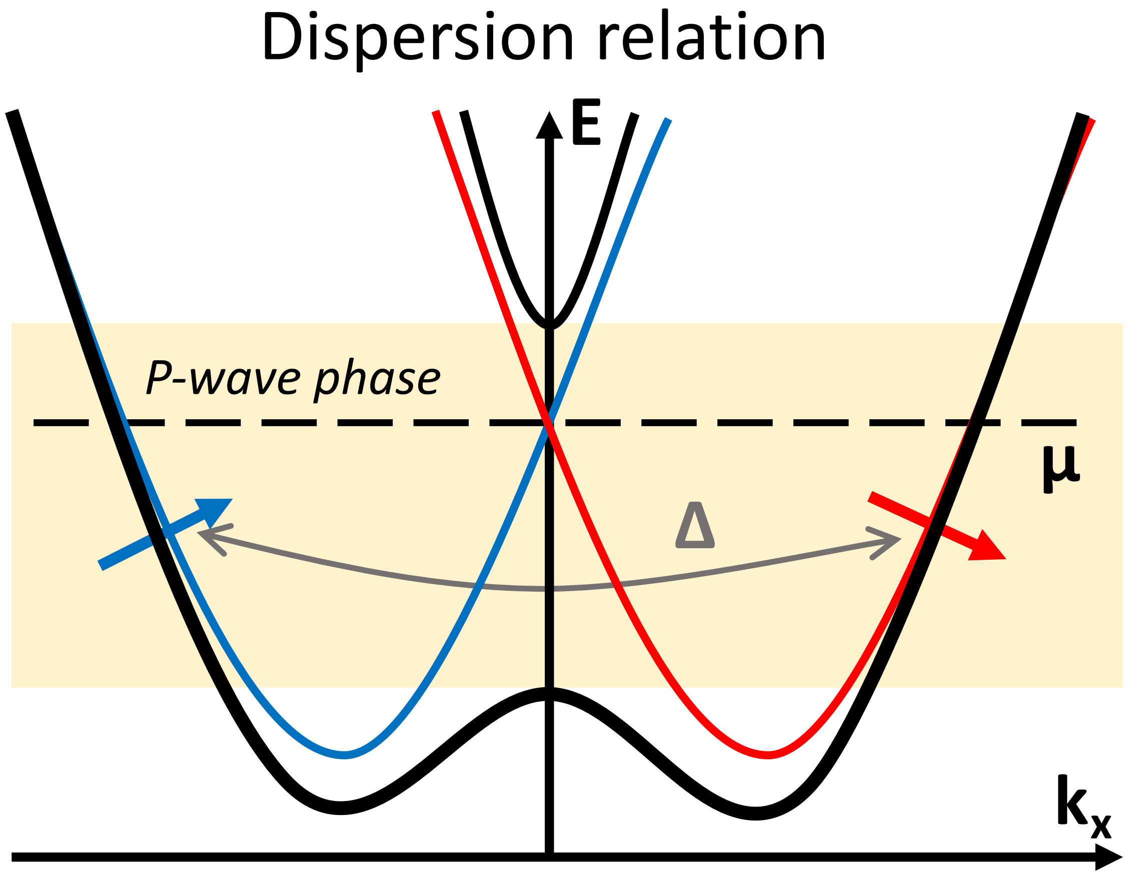




$$\hat{H}_0 = \left[\left(\hbar^2 k_x^2/2m - \mu\right)\sigma_0 + \alpha\sigma_y k_x + V_Z\sigma_x\right]\tau_z$$

- High spin-orbit coupling (Rashba effect) α
- External magnetic field (Zeeman splitting) V_Z
- Induced superconductivity Δ

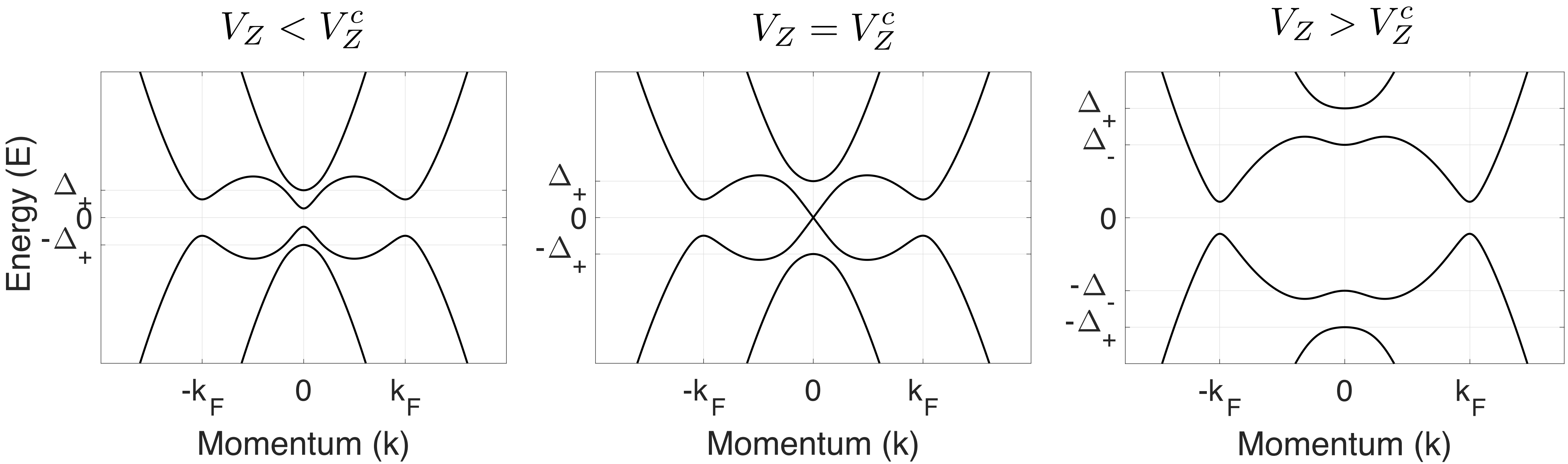




$$\hat{H}_0 = \left[\left(\hbar^2 k_x^2/2m - \mu\right)\sigma_0 + \alpha\sigma_y k_x + V_Z\sigma_x\right]\tau_z + \Delta\sigma_y\tau_y$$

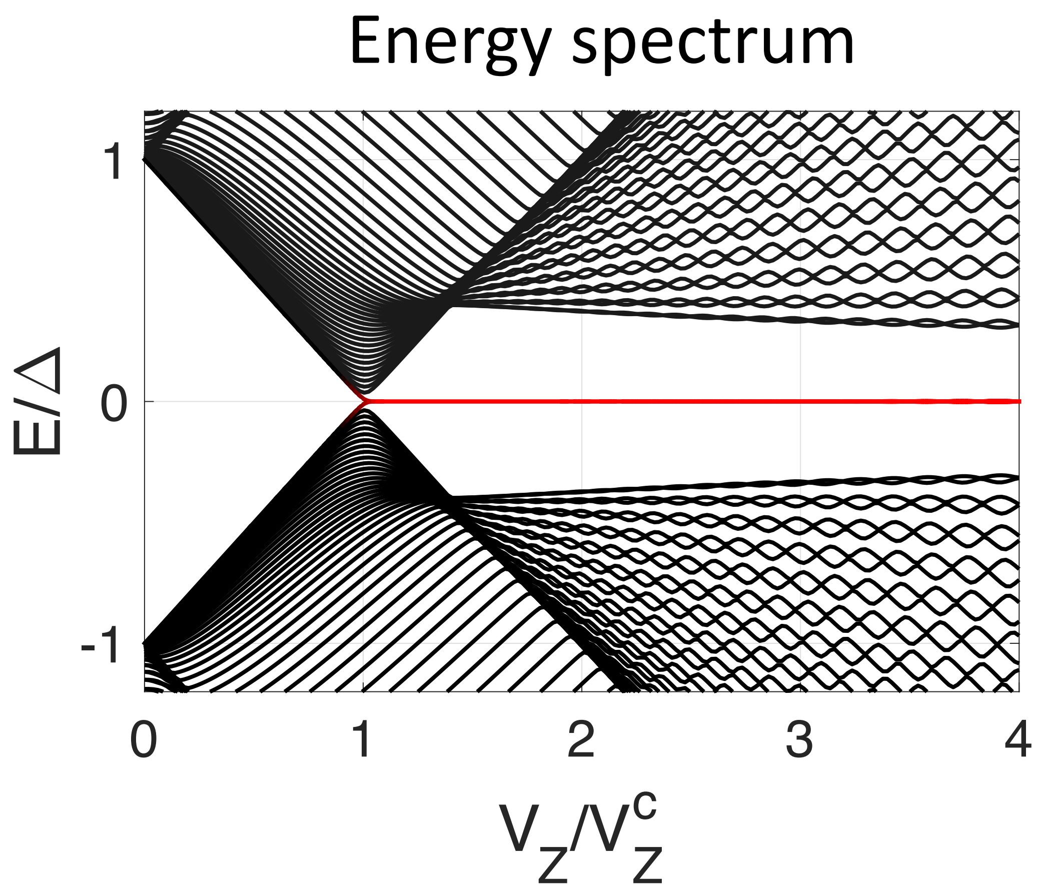
Dispersion relation

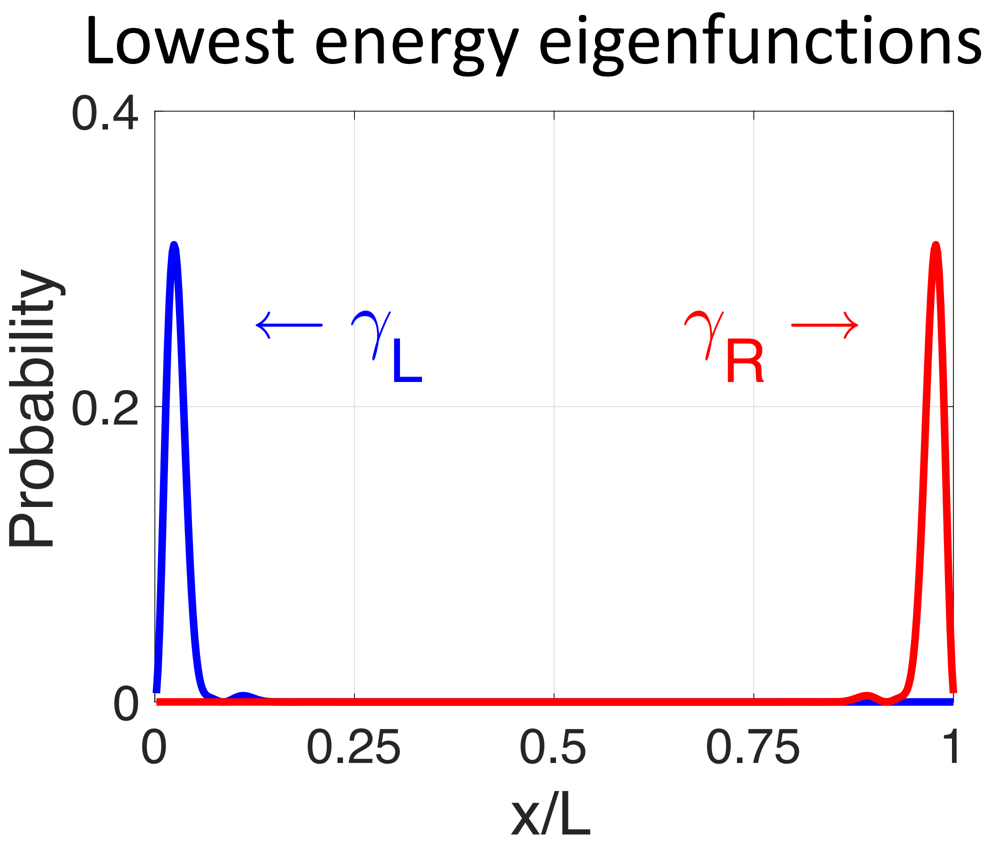
Topological transition: $V_Z^c \equiv \sqrt{\Delta^2 + \mu^2}$



When the gap closes the nanowire undergoes a topological phase transition

Finite long nanowire

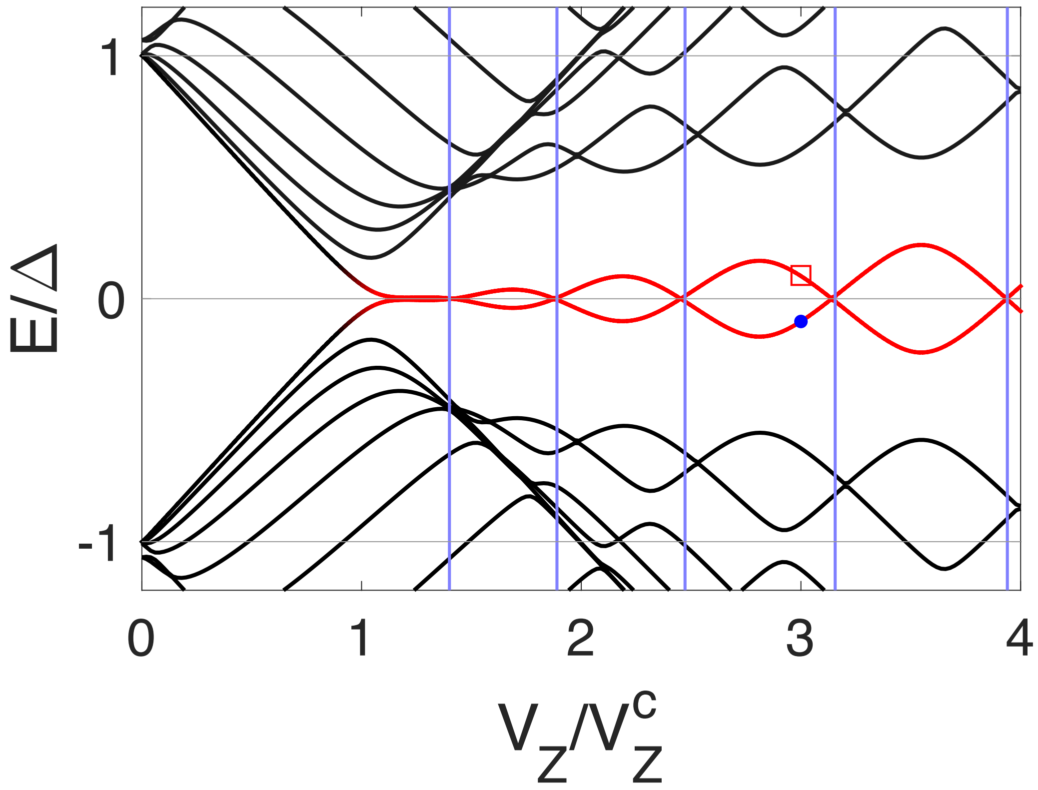




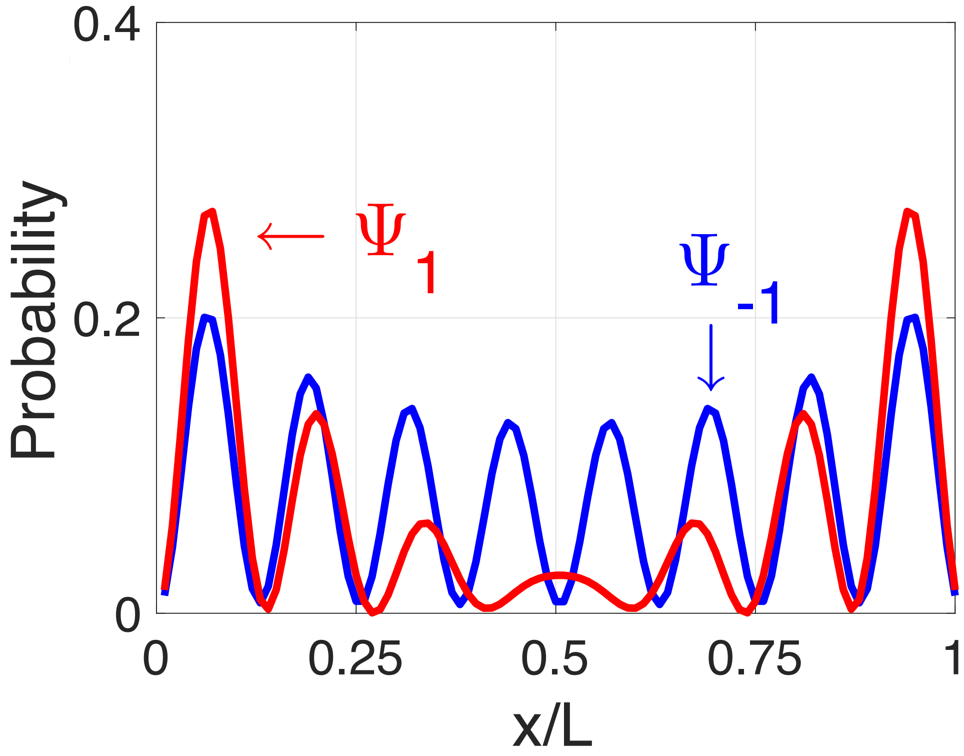
Majoranas emerge as zero energy modes at the edges of the nanowire

Finite short nanowire

Energy spectrum



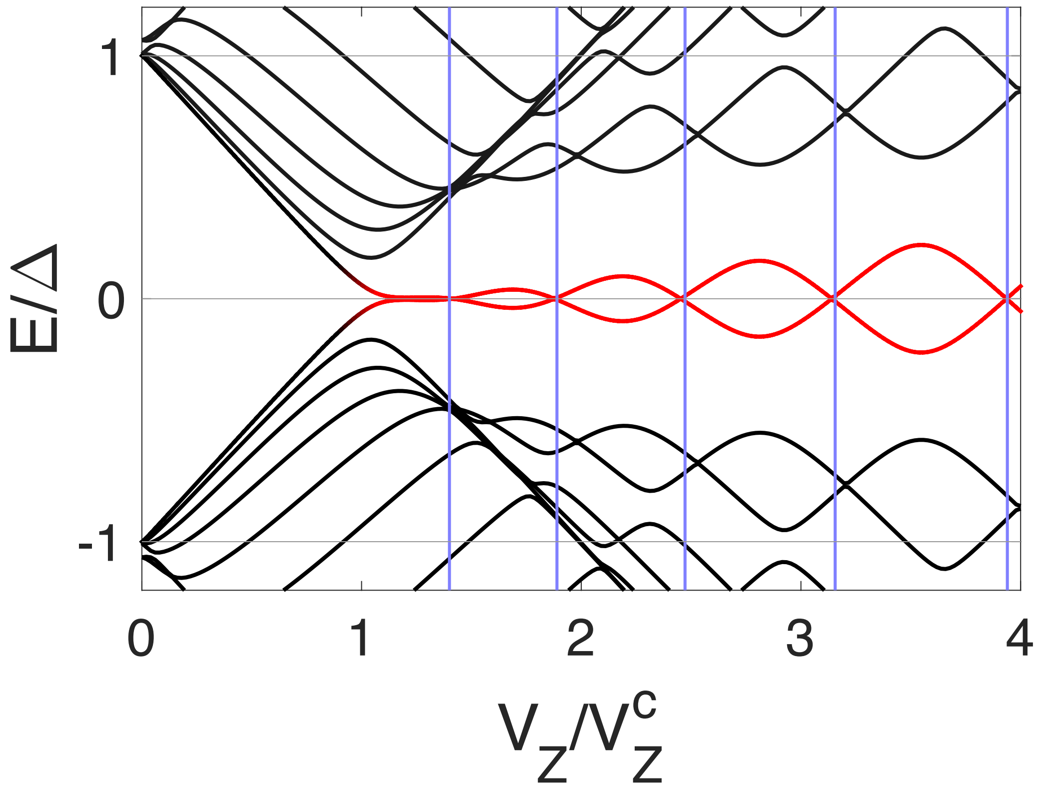
Lowest energy eigenfunctions



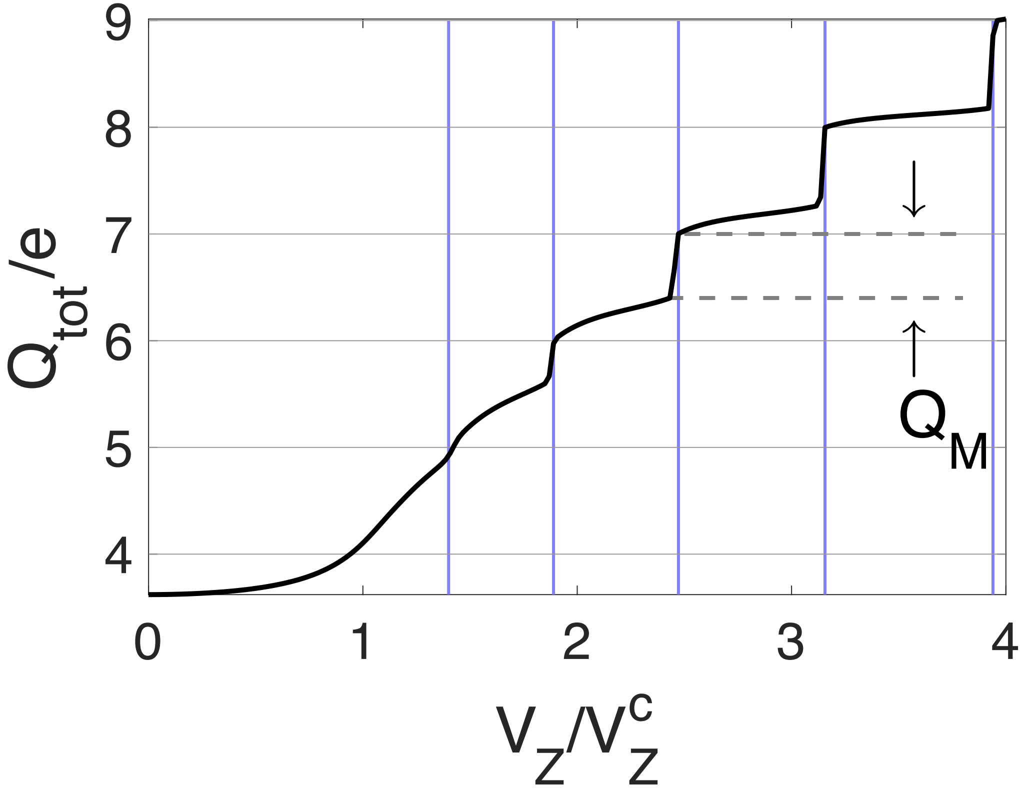
$$\Psi_1 = \Psi_M = (\gamma_L + i\gamma_R)/2 \qquad \Psi_{-1} = \Psi_M^\dagger = (\gamma_L - i\gamma_R)/2$$

Finite short nanowire

Energy spectrum



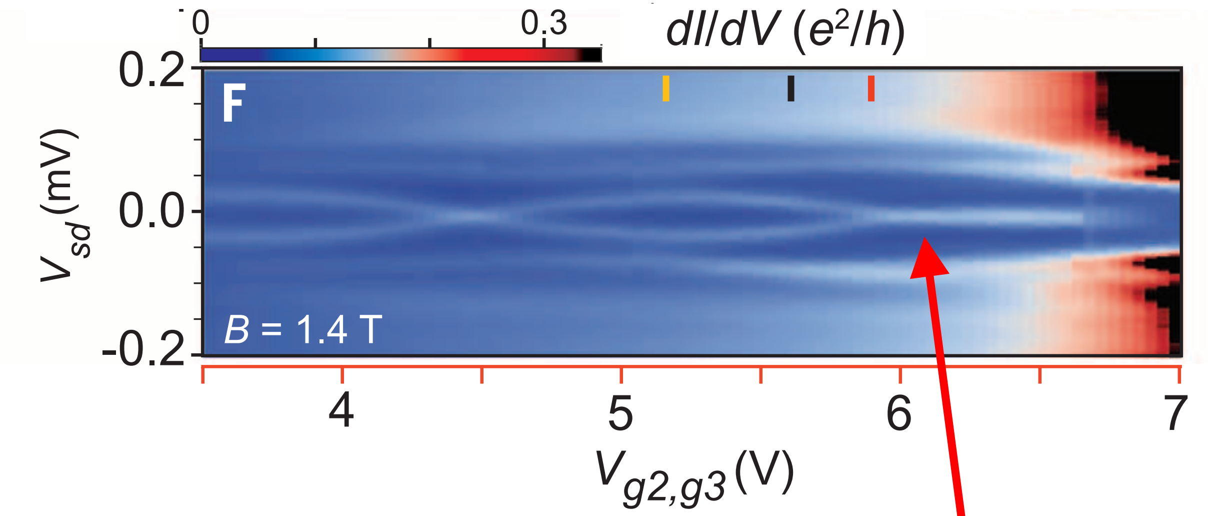
Total charge



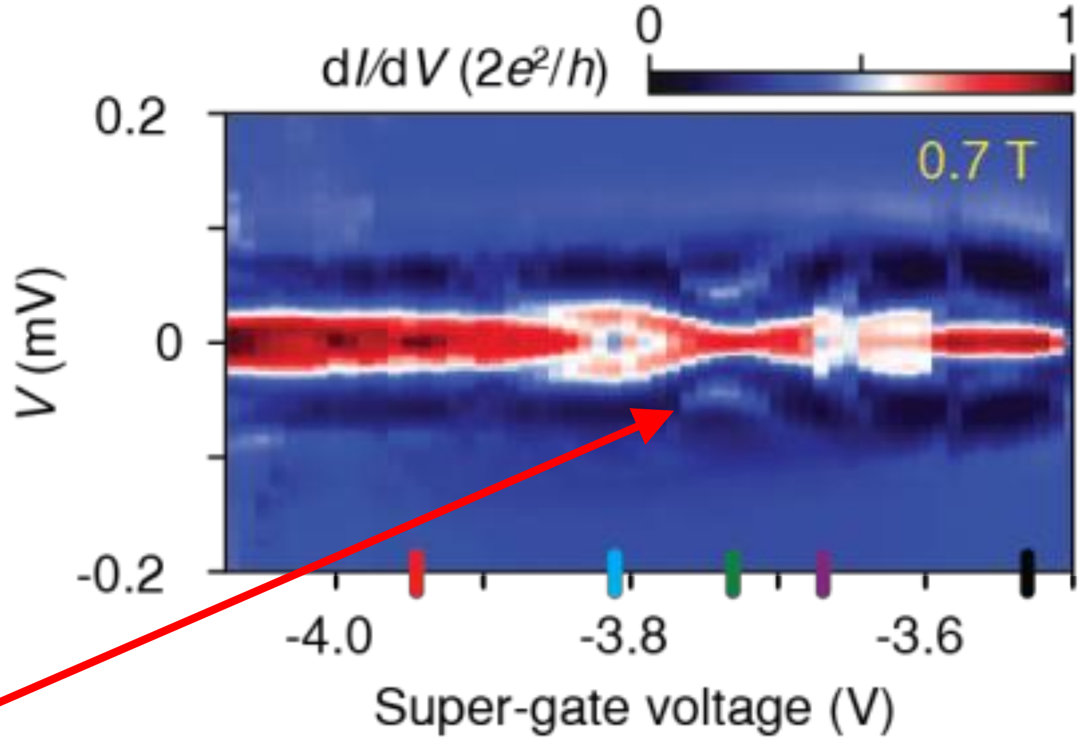
In each parity crossing the total charge increases by an amount Q_M

Experimental measurements: conductance through a Majorana Nanowire

Deng *et al.* Science **354** (2016)



H.Zhang *et al.* arXiv:1710.10701 (2017)

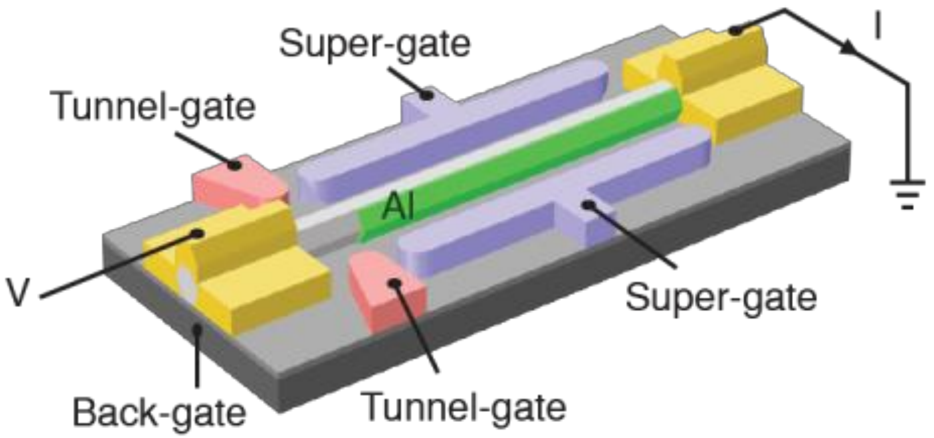


Theory-experiment disagreement

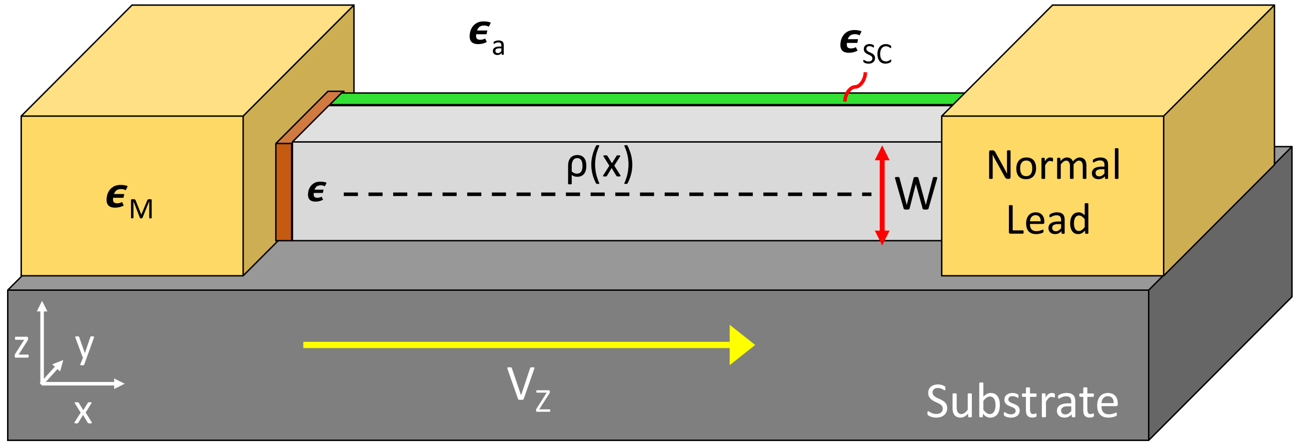
- Zero-energy pinned regions
- Non-topological energy levels approaching zero energy

Electrostatic environment

Experimental set up of
Deng *et al.* Science **354** (2016)



Model of the electrostatic environment



- InSb Nanowire: $\epsilon = 17{,}7$
- Vacuum: $\epsilon_a \simeq 1$
- SC shell: $\epsilon_{SC} \simeq 100$
- SiO_2 substrate: $\epsilon_d = 3{,}9$
- Normal leads: $\epsilon_M \to \infty$
- Nanowire $\begin{cases} L = 1\mu\text{m} \\ R = 50\text{nm} \end{cases}$

Electrostatic environment

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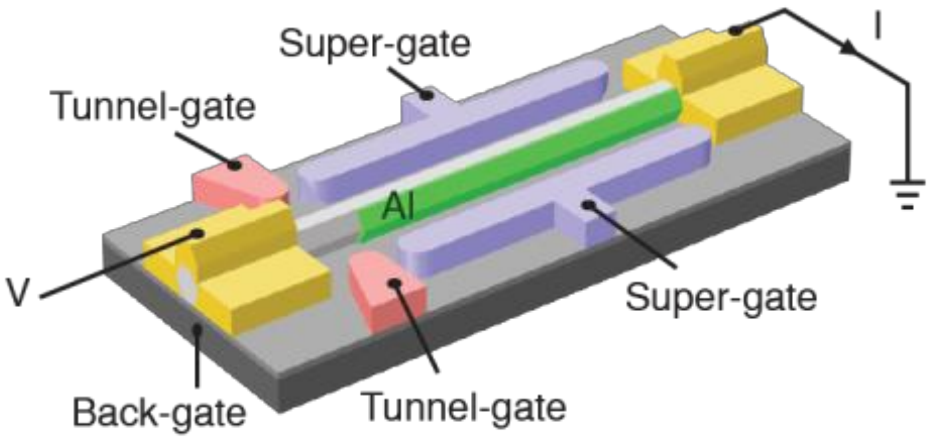
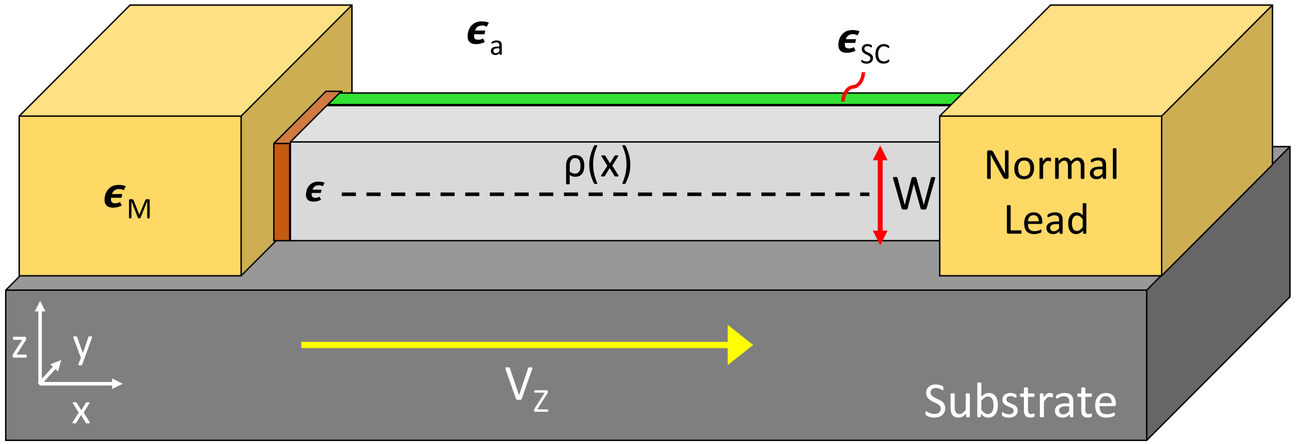
Deng *et al.* Science **354** (2016)



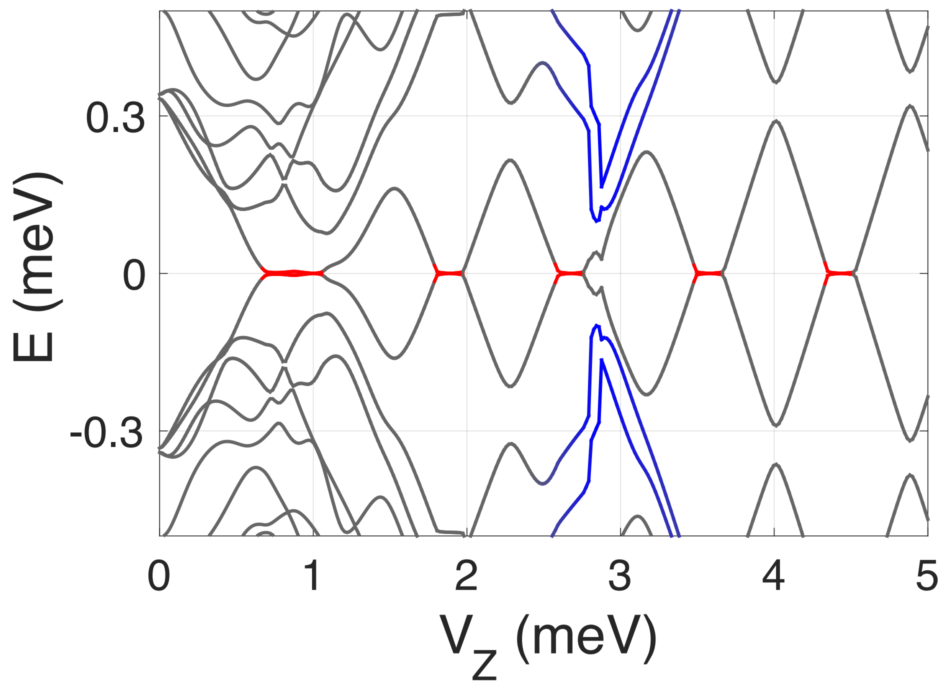
Model of the electrostatic environment



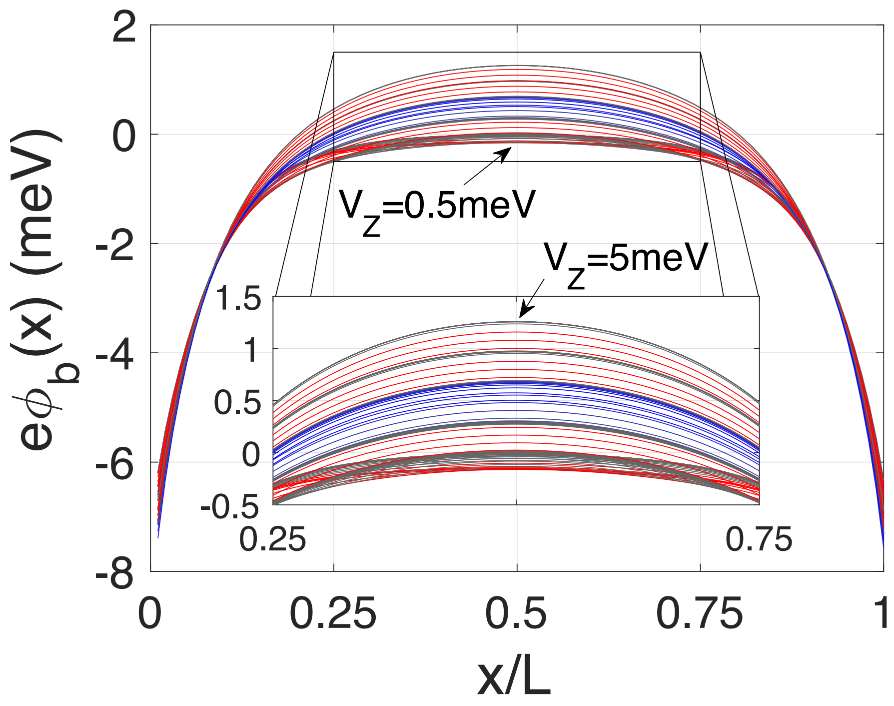
$$\hat{H} = \hat{H}_0 + e\phi_b(x)\,\sigma_0\tau_z \longrightarrow \phi_b(x) = \int dx' V_b(x,x')\,\langle\hat{\rho}(x')\rangle$$

Results

Energy spectrum



Bound charges electrostatic potential

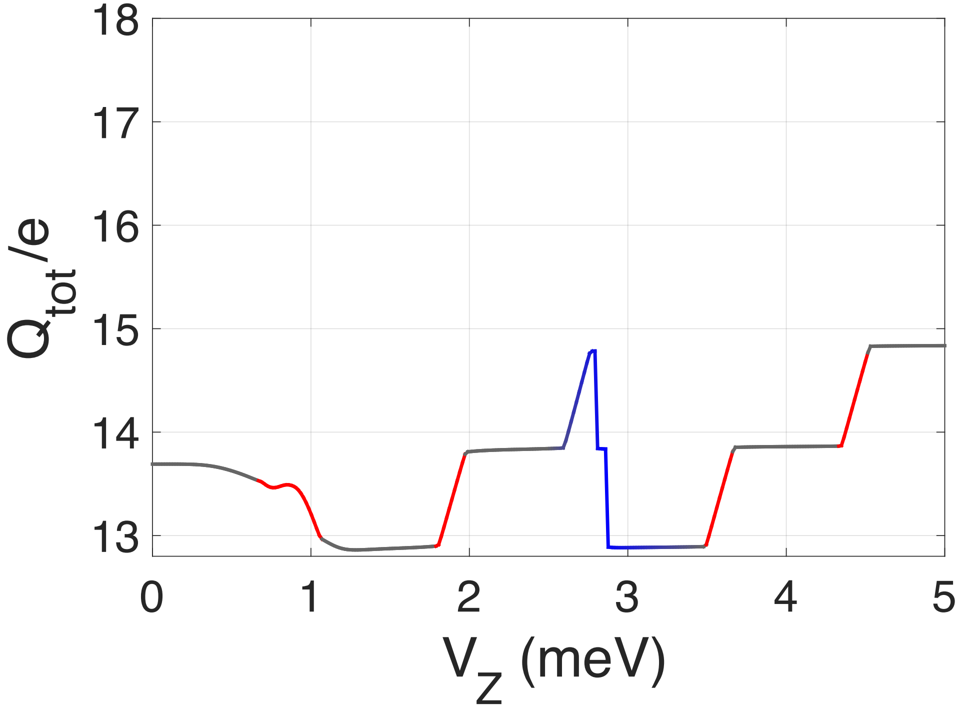


$$m = 0{,}015 m_e$$
$$\alpha = 20\text{meV} \cdot \text{nm}$$
$$\Delta = 0.3\text{meV}$$
$$\mu = 0{,}5\text{meV}$$

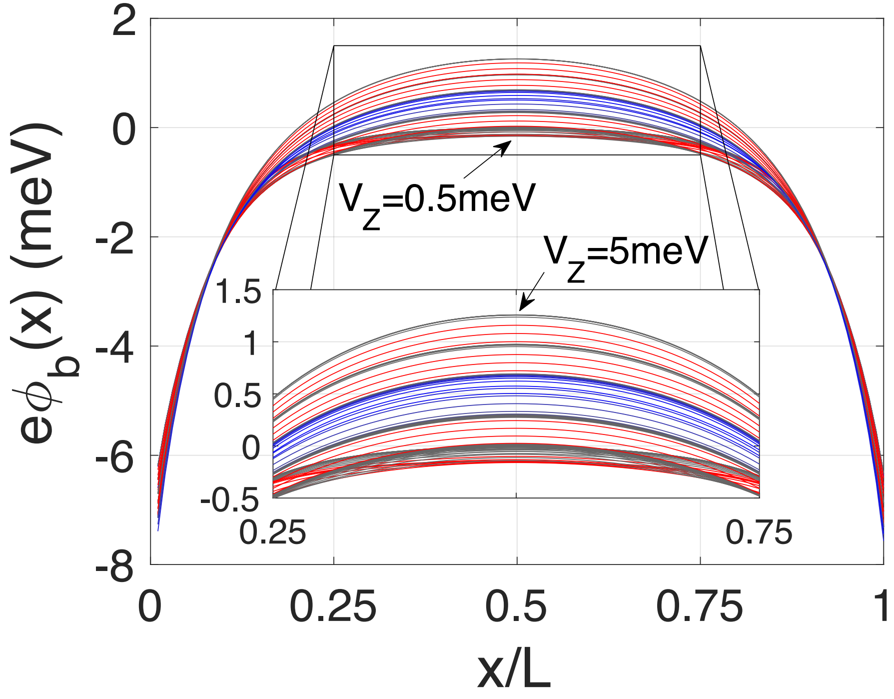
The potential is repulsive at the nanowire middle, while it is attractive at the nanowire edges

Repulsive interaction

Total charge



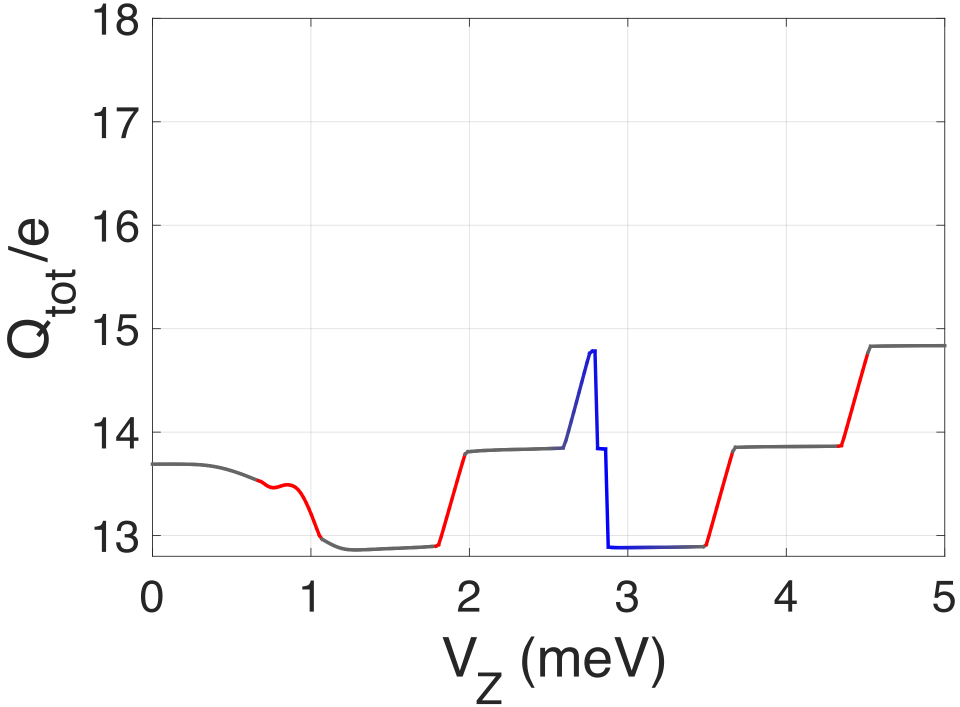
Bound charges electrostatic potential



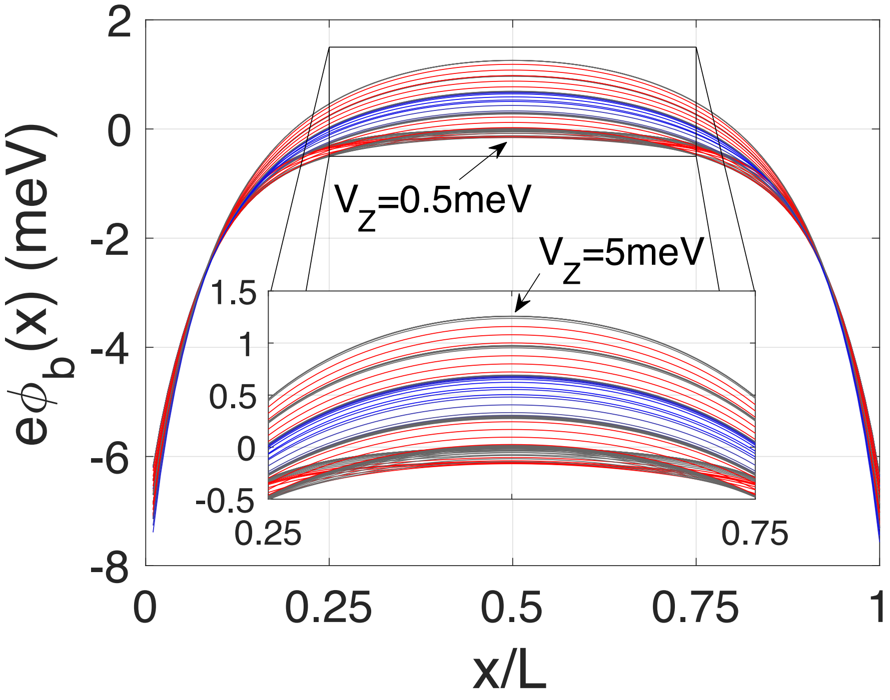
Because of the repulsive part, charge enters into the nanowire progressively (instead of by jumps)

Repulsive interaction

Total charge



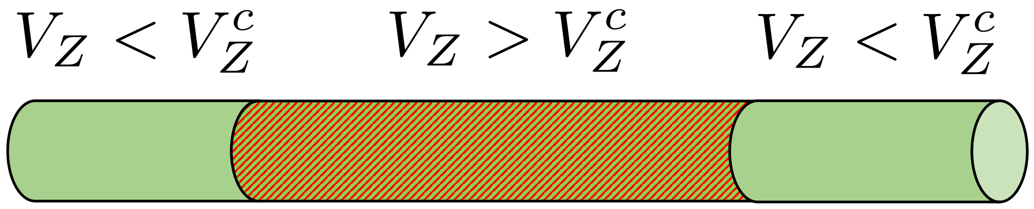
Bound charges electrostatic potential



It freezes the Majorana modes, leading to zero energy pinned regions

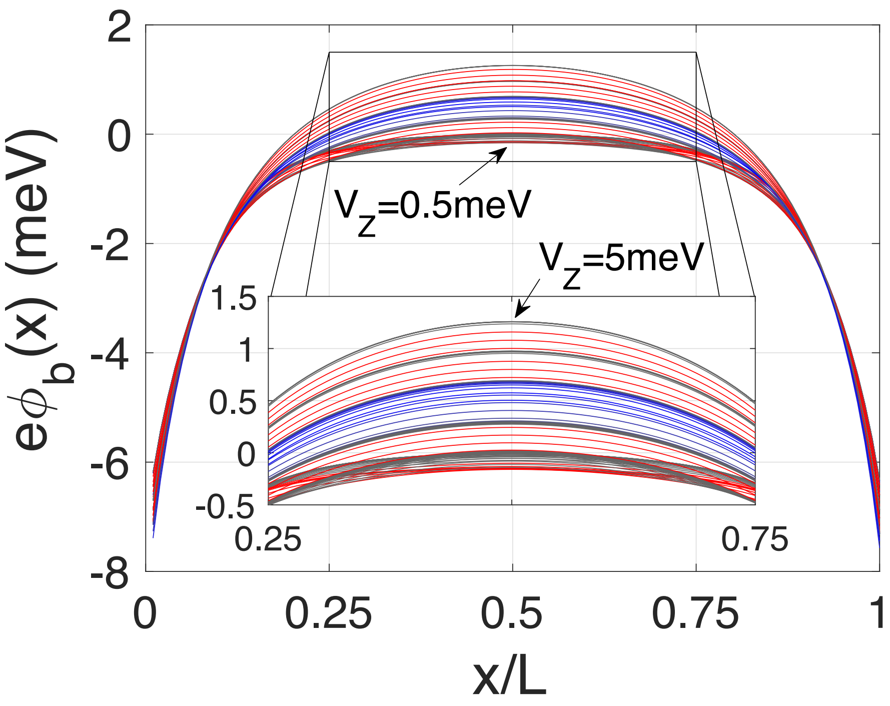
Atractive interaction

Topological phase along the nanowire



Where: $V_Z^c\left(x, V_Z\right) = \sqrt{\left(\mu - \phi_b\left(x\right)\right)^2 + \Delta^2}$

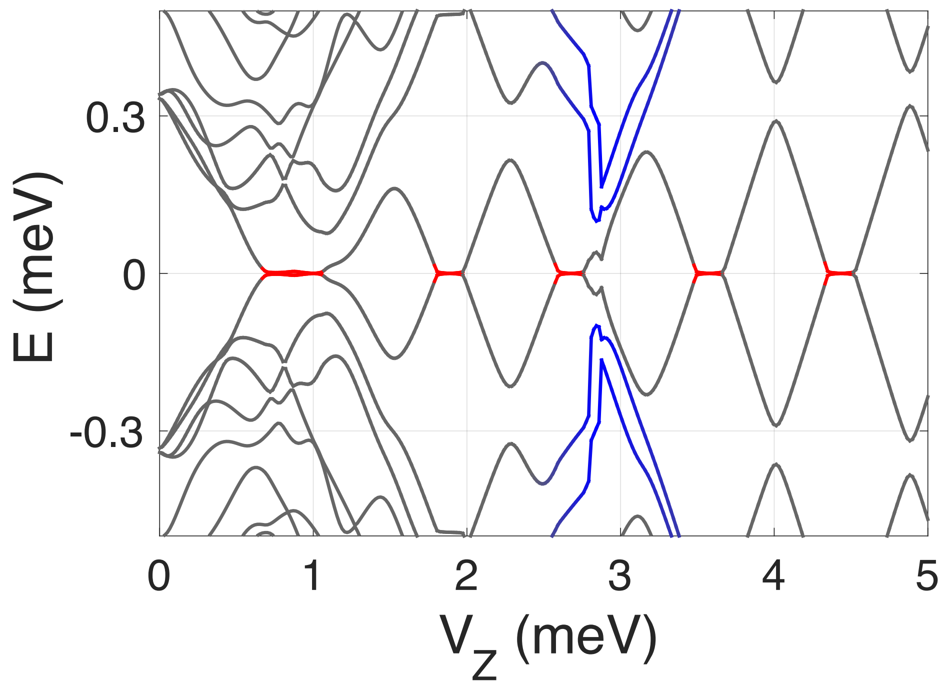
Bound charges electrostatic potential



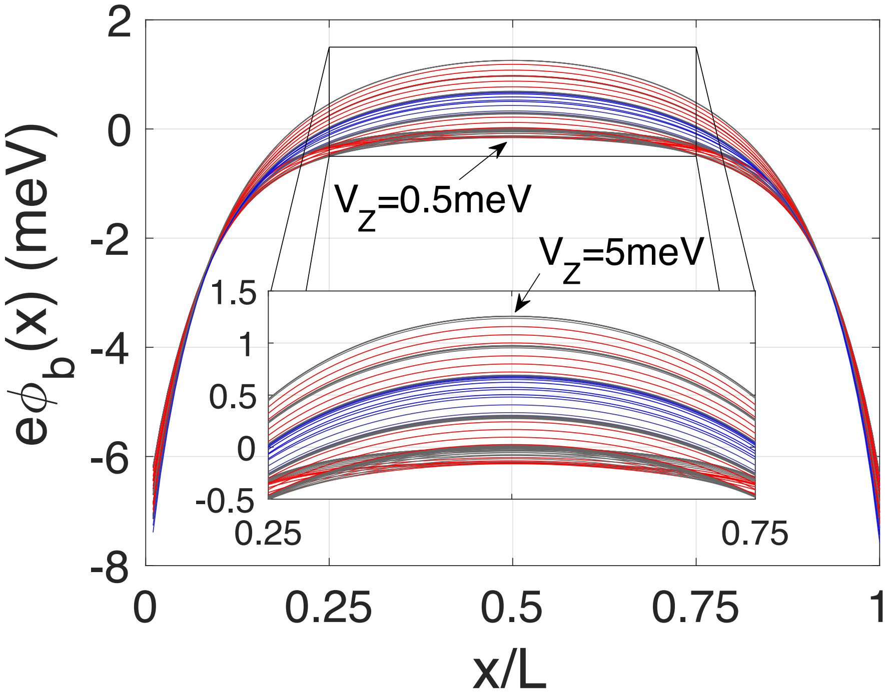
Because of the attractive part, the nanowire undergoes the topological phase by regions

Atractive interaction

Energy spectrum



Bound charges electrostatic potential



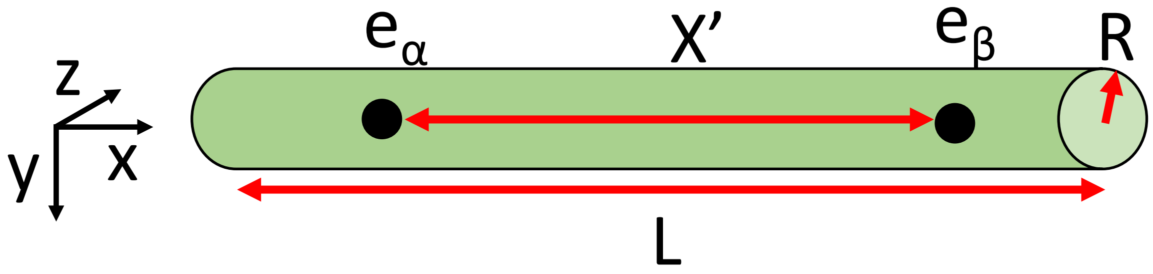
It builds two Quantum Dots at each end of the nanowire which hybridize with the Majoranas

3. Conclusions

- The interaction with the electrostatic environment of the nanowire could explain some discrepancies between theory and experiments.
- The repulsive part of the electrostatic interaction makes Majoranas more stable under electrostatic and magnetic perturbations.
- Quantum dots are naturally built at the edges of these nanowires due to the attractive interaction created by the leads.
- Both features could help control Majorana qubits, which can be used as building blocks in quantum computation.

Supplementary material for questions

A. Electron-Electron interaction inside the nanowire



- Electron-Electron interaction in the Thomas-Fermi limit:

$$\hat{V}_{e-e} = \check{c}^\dagger_\alpha \check{c}_\alpha V^{TF}_{\alpha\beta} \check{c}^\dagger_\beta \check{c}_\beta \longrightarrow V^{TF}(x') = \frac{\sqrt{\pi}}{4\pi\epsilon\epsilon_0 R} e^{x'^2/R^2 - |x'|/\lambda_{TF}} \mathrm{Erfc}\left(\frac{|x'|}{R}\right)$$

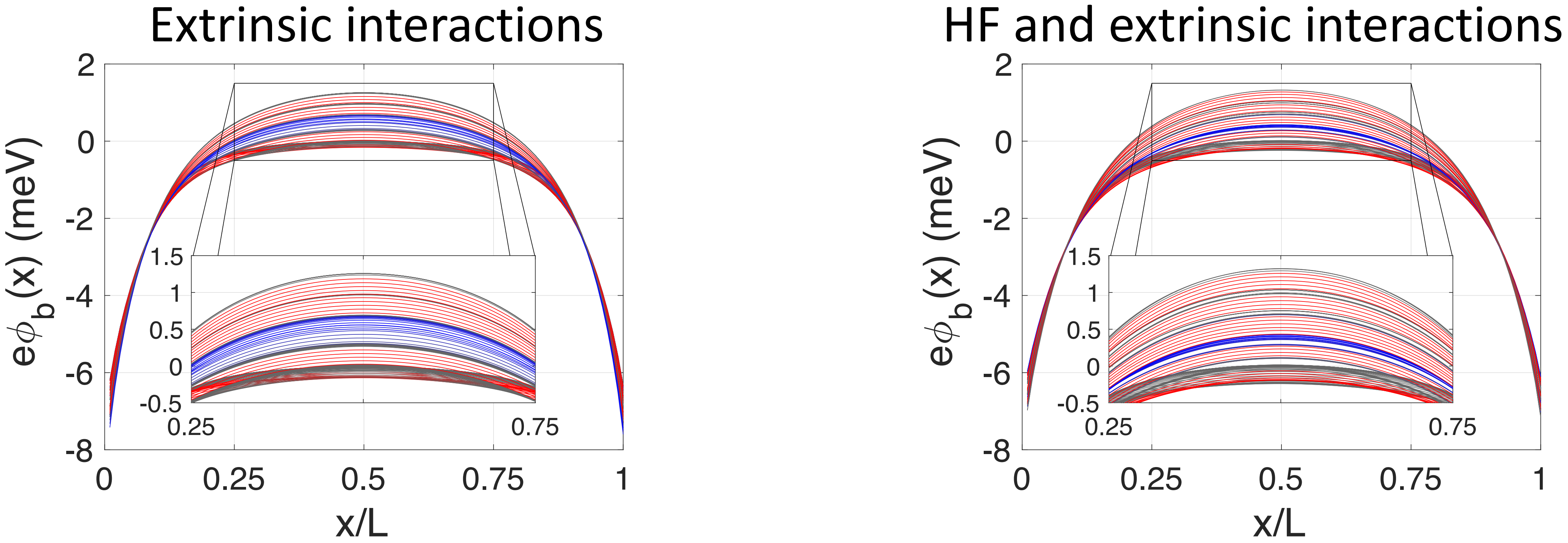
- Wick's theorem:

$$\hat{V}_{eff} = V^{TF}_{\alpha\beta}\Big[\underbrace{\langle \check{c}^\dagger_\alpha \check{c}_\alpha \rangle \check{c}^\dagger_\beta \check{c}_\beta + \left\langle \check{c}^\dagger_\beta \check{c}_\beta \right\rangle \check{c}^\dagger_\alpha \check{c}_\alpha}_{\textit{Hartree}} + \underbrace{\langle \check{c}^\dagger_\alpha \check{c}_\beta \rangle \check{c}^\dagger_\beta \check{c}_\alpha + \left\langle \check{c}_\alpha \check{c}^\dagger_\beta \right\rangle \check{c}^\dagger_\alpha \check{c}_\beta}_{\textit{Fock}} - \underbrace{- \left\langle \check{c}^\dagger_\alpha \check{c}^\dagger_\beta \right\rangle \check{c}_\alpha \check{c}_\beta - \langle \check{c}_\alpha \check{c}_\beta \rangle \check{c}^\dagger_\alpha \check{c}^\dagger_\beta}_{\textit{Bogoliubov}}\Big]$$

- Nambu structure:

$$\hat{V}^{HFB} = \frac{1}{2}\check{c}^\dagger\left(\hat{V}_{eff} - \Lambda \hat{V}^t_{eff}\Lambda\right)\check{c} \longrightarrow \Lambda \equiv \tau_x \otimes \mathbb{I}^{\mathrm{site}}_{N\times N} \otimes \sigma_0$$

Electrostatic potential

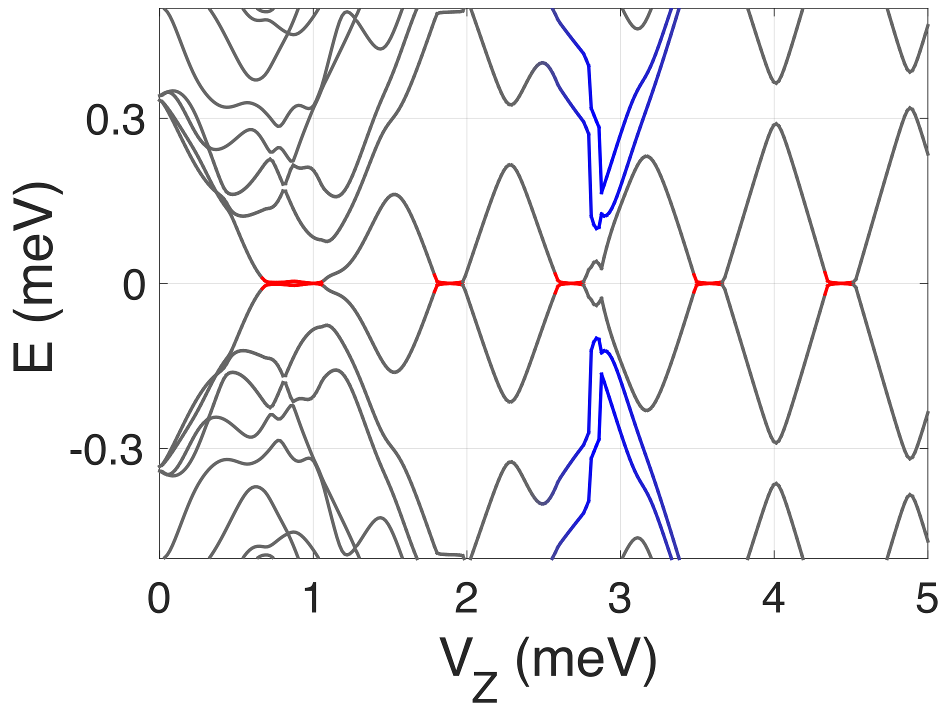


$\lambda_{TF} = 10\text{nm}$

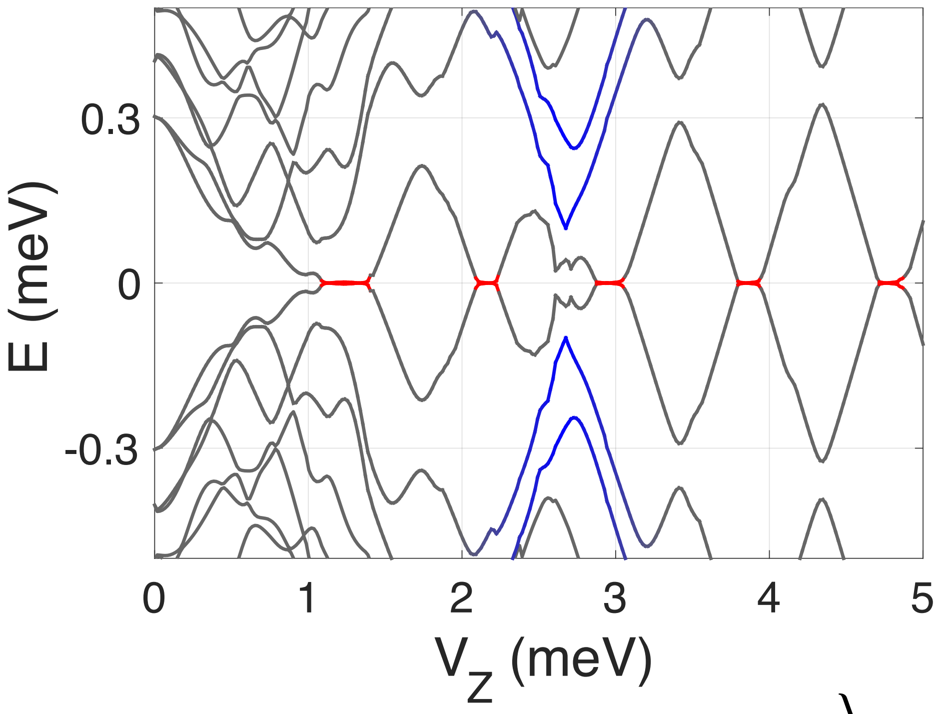
The bound charges electrostatic potential is (a little bit) flatter

Energy spectrum

Extrinsic interactions



HF and extrinsic interactions

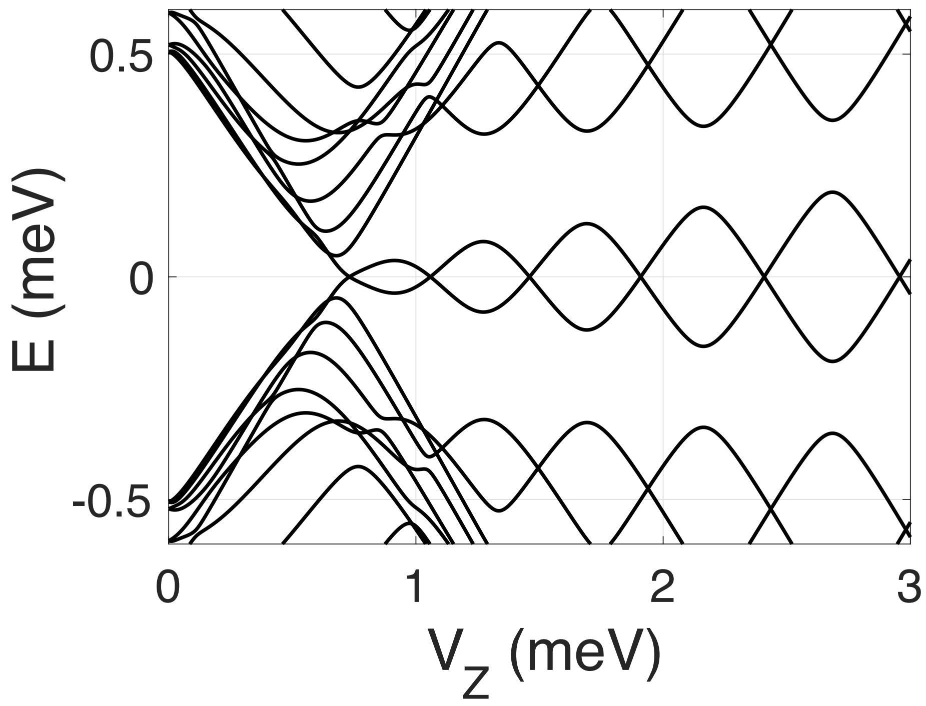


$\lambda_{TF} = 10\text{nm}$

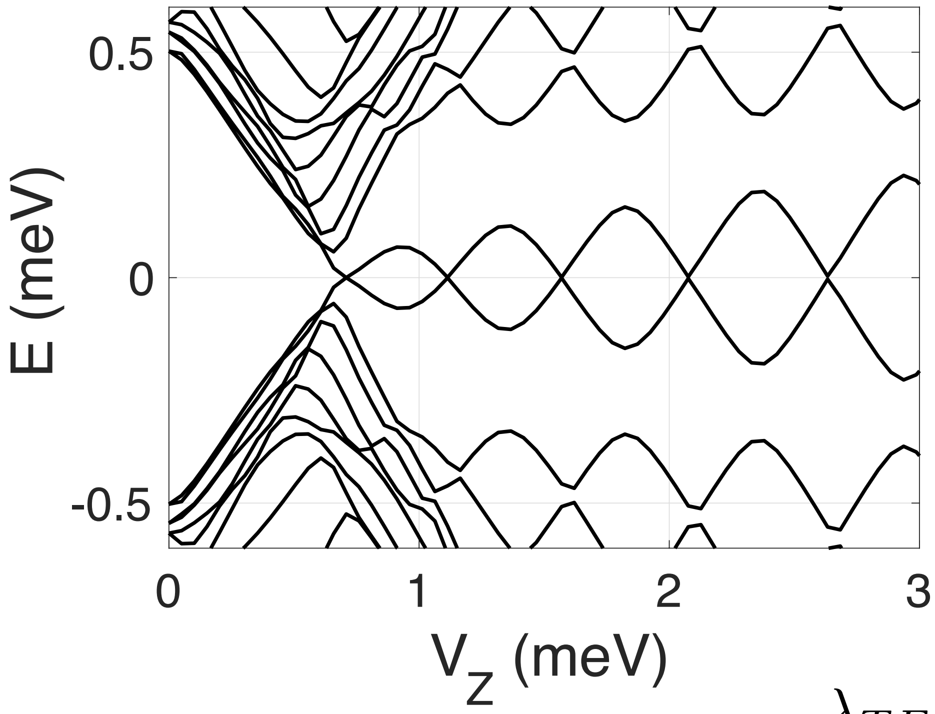
Features (pinning and QDELs) are not destroyed

Energy spectrum

No e-e interactions



Hartree-Fock e-e interactions



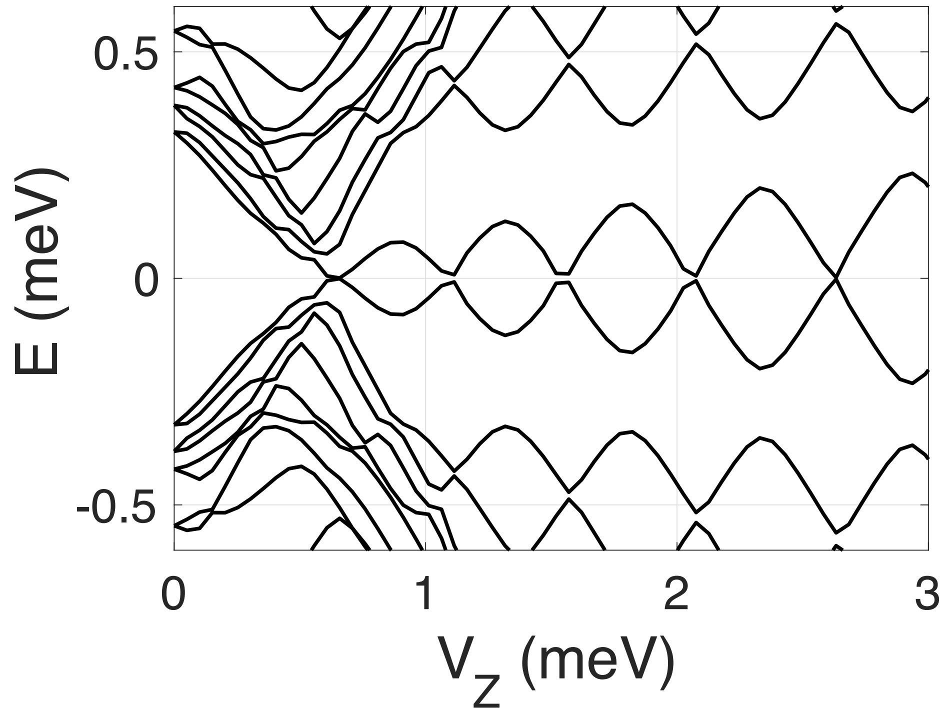
$\lambda_{TF} = 10\mathrm{nm}$

HF interaction changes the chemical potential and the Zeeman splitting

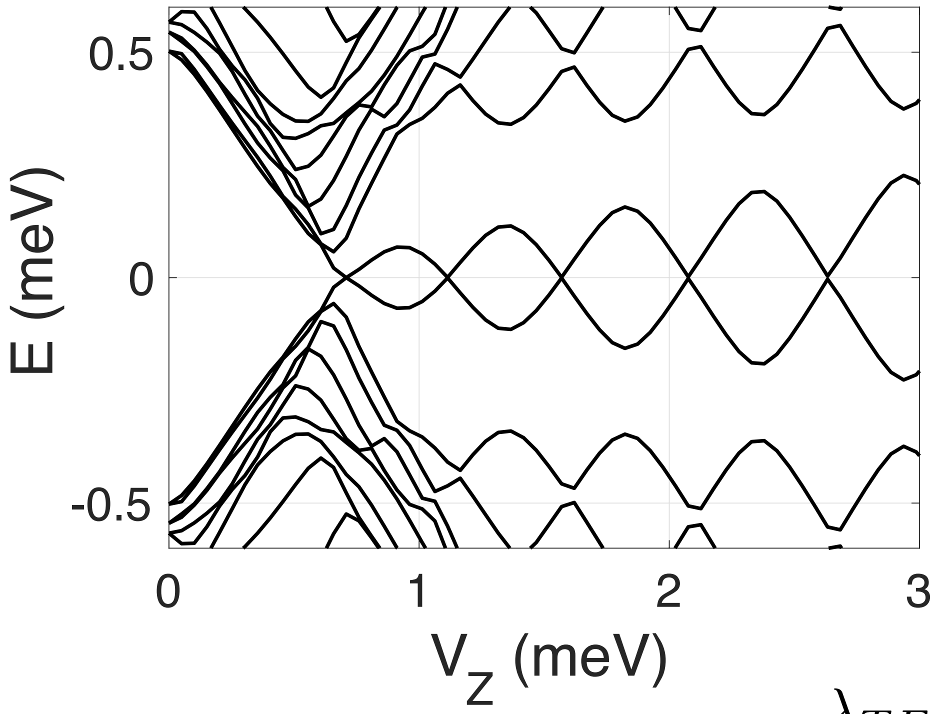
A. Electron-Electron interaction inside the nanowire

Energy spectrum

Hartree-Fock-Bogoliubov e-e interactions



Hartree-Fock e-e interactions

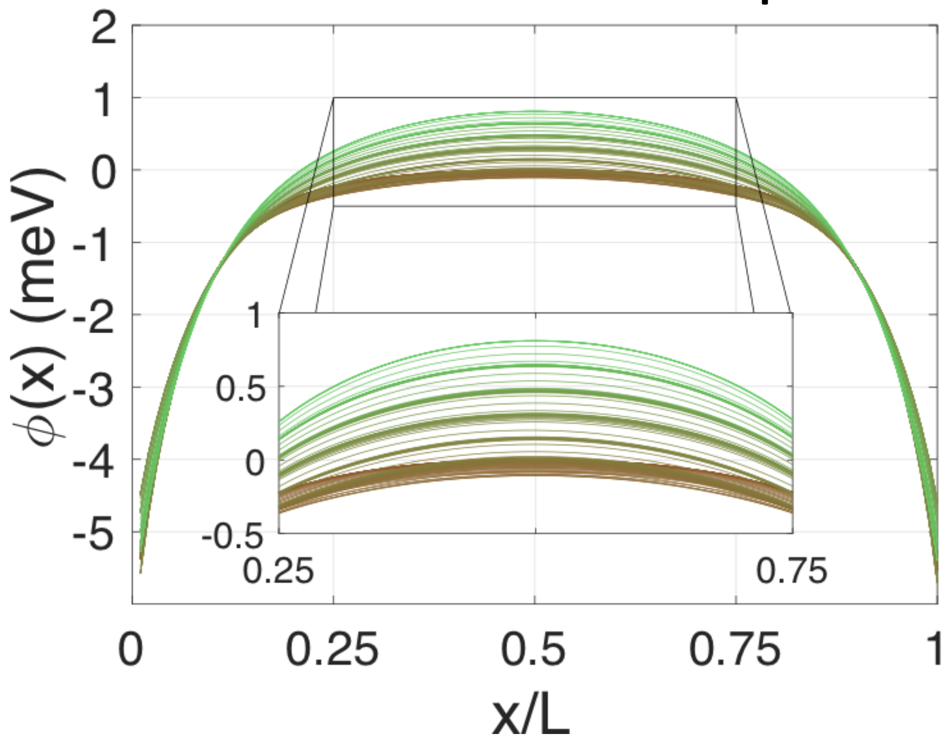


$\lambda_{TF} = 10\mathrm{nm}$

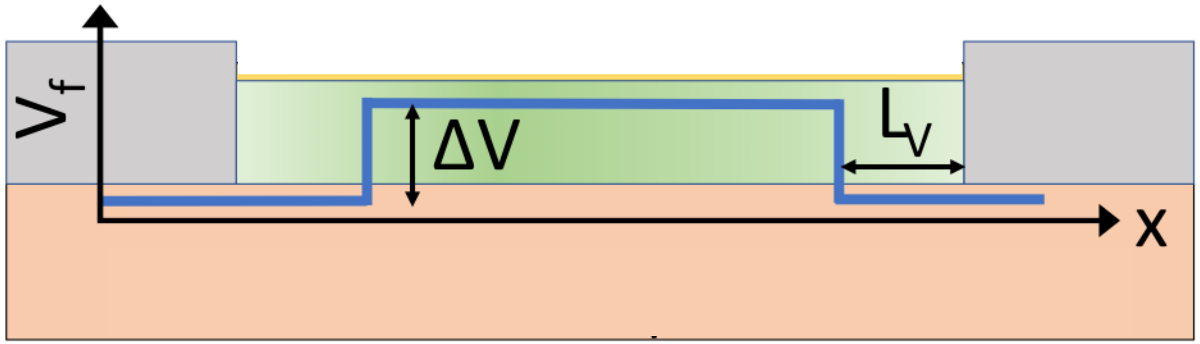
HFB interaction changes also the induced superconductor gap

QD-Majorana nanowire model

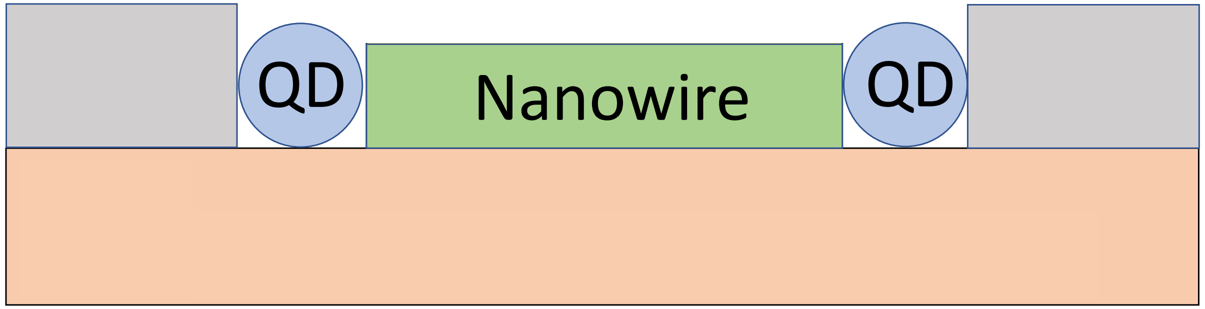
Full model electrostatic potential



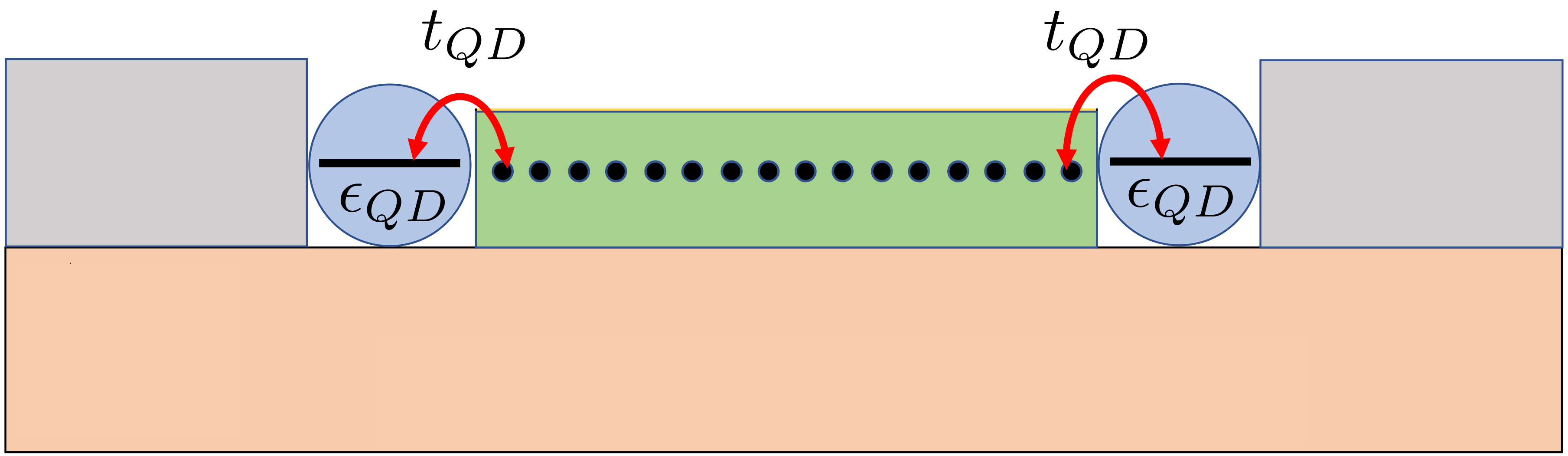
Fixed electrostatic potential model



QDs-Majorana nanowire model



QD-Majorana nanowire model



$$\hat{H}_{QD-w} = \hat{H}_{QD}\tau_z + \hat{H}_{hopping}\tau_z + \hat{H}_0 \rightarrow$$

$$\rightarrow \begin{cases} \hat{H}_{QD} = d_\sigma^\dagger \left(\epsilon_{QD}\sigma_0 + V_Z\sigma_z\right) d_\sigma + U n_\uparrow n_\downarrow \\ \hat{H}_{hop} = t_{QD}\left(c_{0\sigma}d_\sigma^\dagger + c_{N+1,\sigma}d_\sigma^\dagger + \text{h.c.}\right) \end{cases}$$

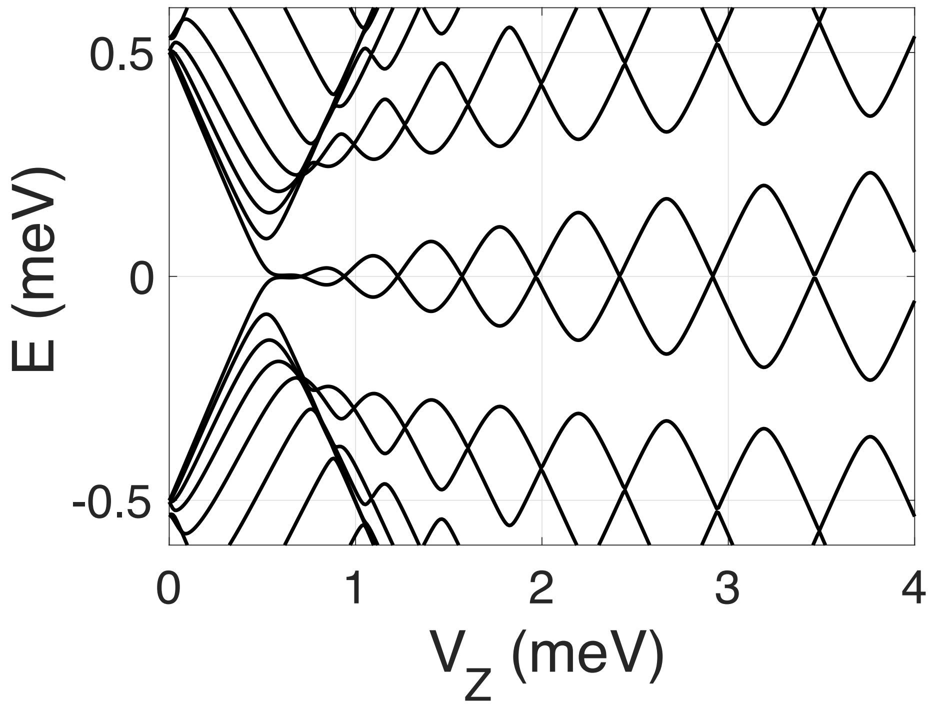
$$U = 3\text{meV}$$

$$t_{QD} = t$$

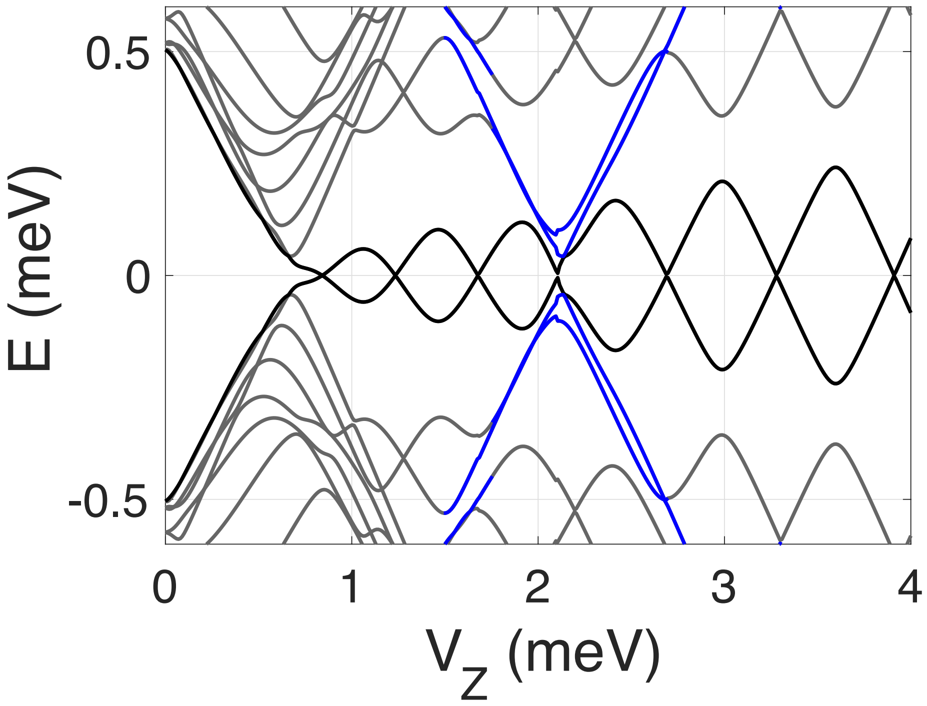
$$\epsilon_{QD} = 19\text{meV}$$

QD-Majorana nanowire model

Energy spectrum (non-interacting)

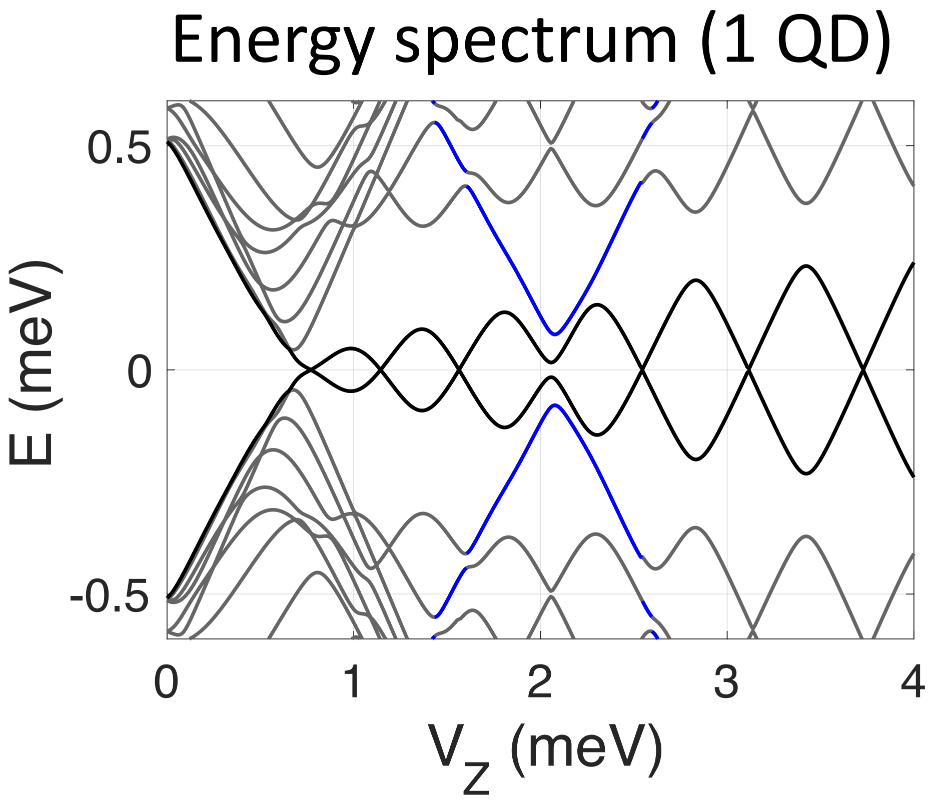


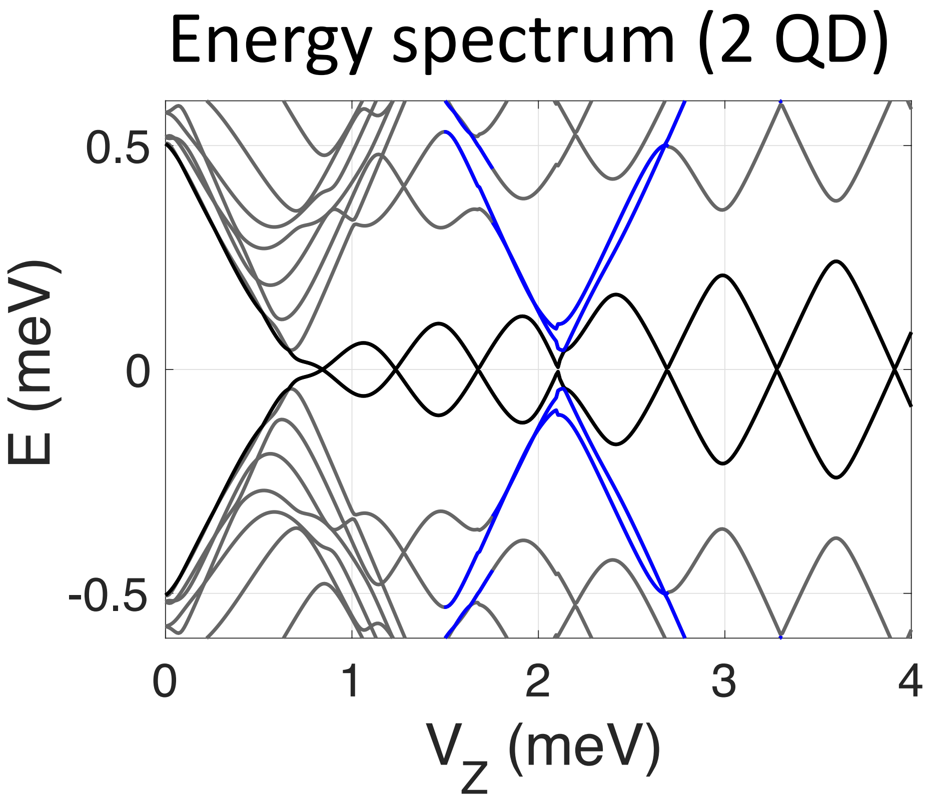
Energy spectrum (QD-MN model)



The energy levels of the QDs anticross MZM energies

QD-Majorana nanowire model



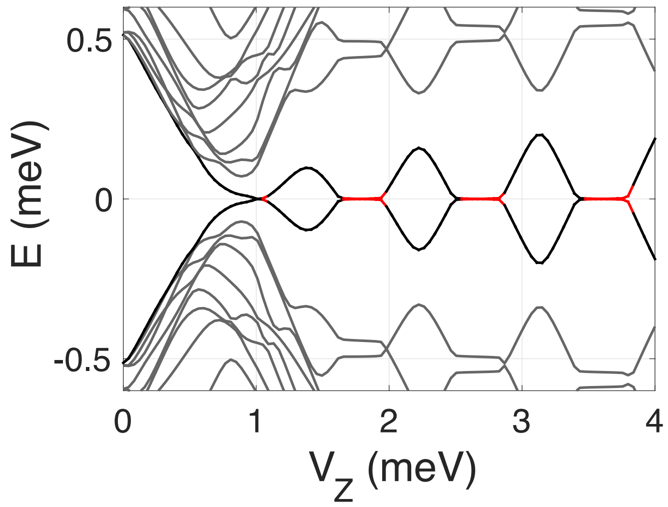


There are four QDELs because there are two QDs

C. Majorana modes in quantum computation

A Majorana qubit is:

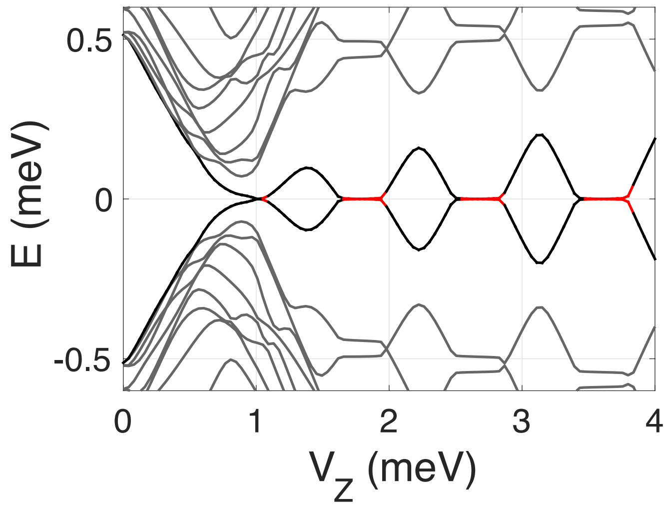
- A doubly degenerate ground state, far enough from the rest of the energy levels:
  - Sub-gap states Majorana Zero Energy Modes → Pinning

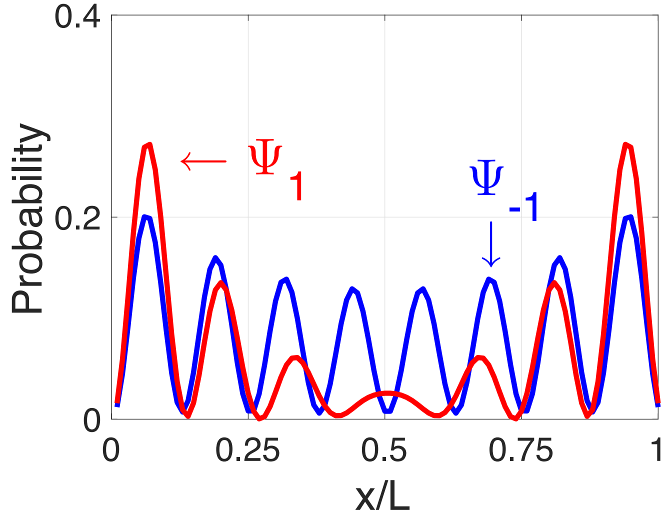


C. Majorana modes in quantum computation

A Majorana qubit is:

- A doubly degenerate ground state, far enough from the rest of the energy levels:
  - Sub-gap states Majorana Zero Energy Modes → Pinning
- Robust against sources of decoherence:
  - Non-local wave-function → Quantum Dots
  - Non-Abelian statistics (non-trivial topology)

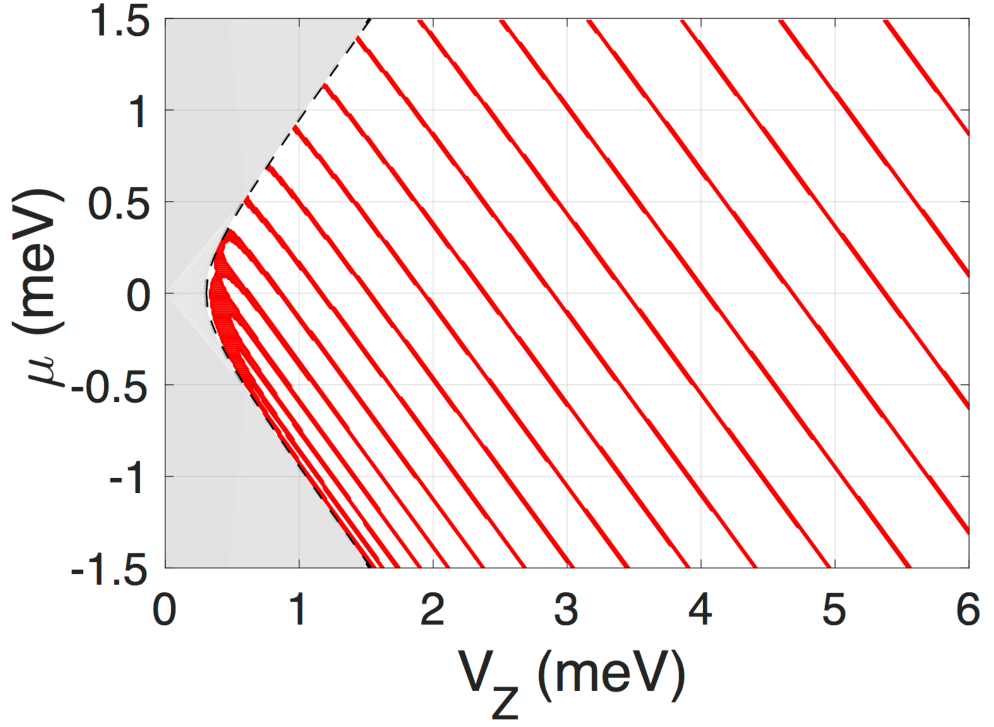




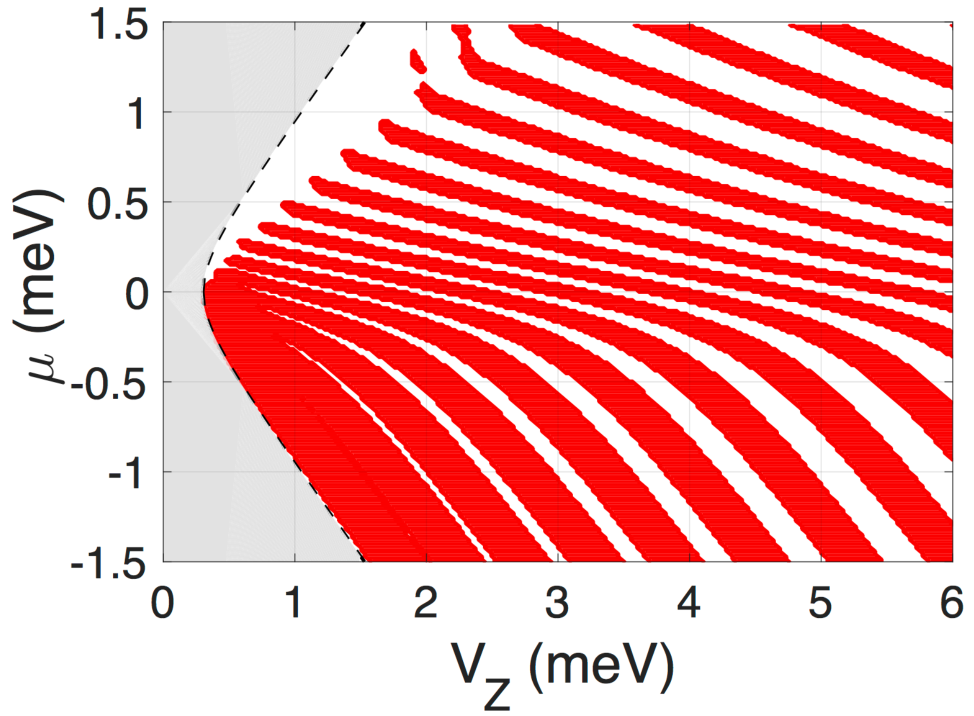
C.Nayak et *al. Rev. Mod. Phys.* **80** (2008)

Pinned regions for different environments

Pinning (non-interacting)



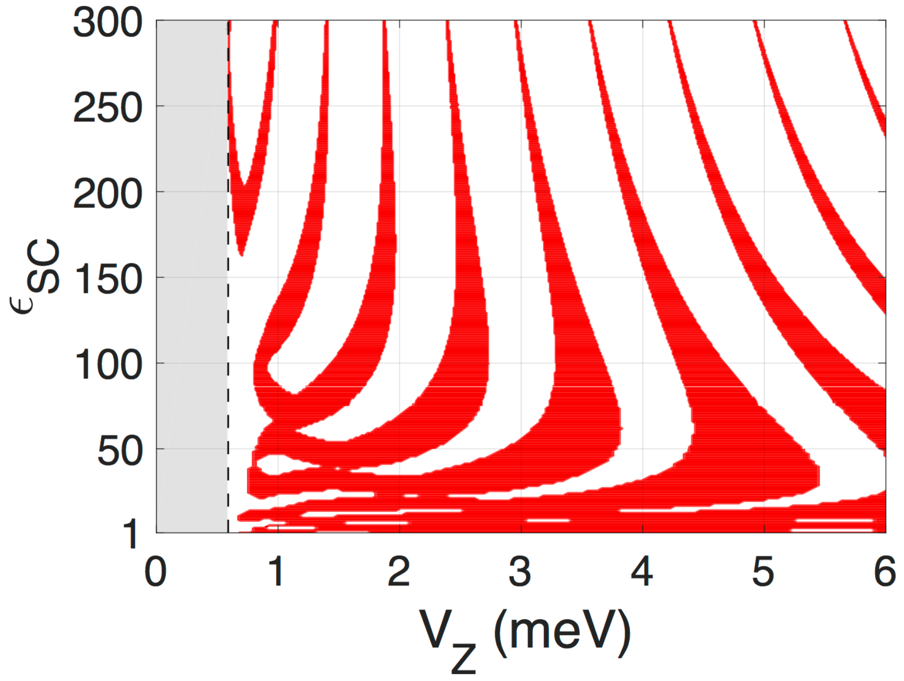
Pinning (interacting)



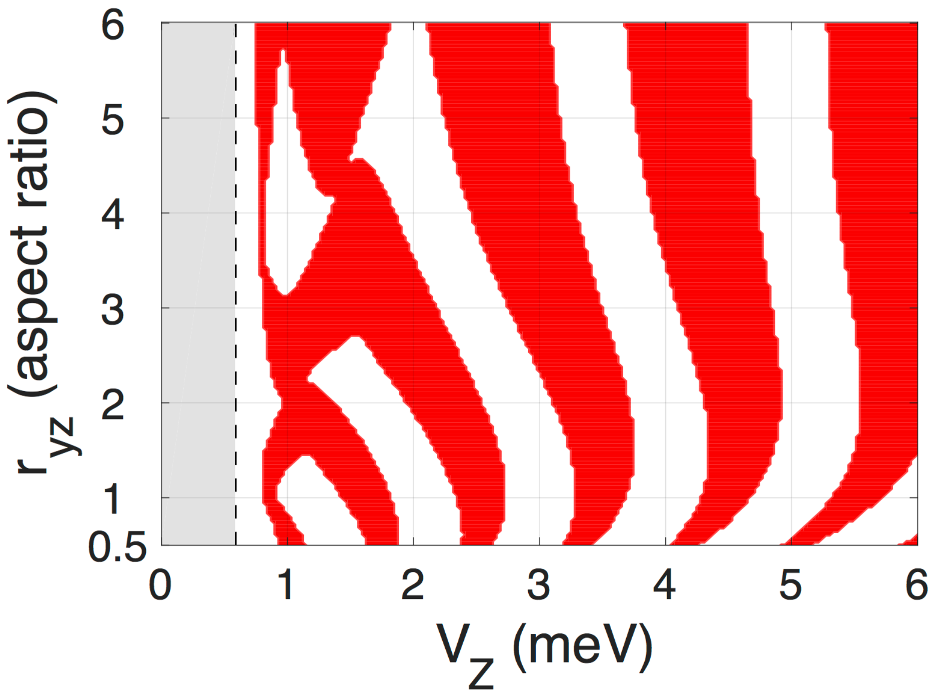
Pinning is general for all chemical potentials

Pinned regions for different environments

Different SC permittivities



Different nanowire radius



Pinning is not general for all kind of environments

E. Equations

Interaction ignoring the leads:

$$V_b(x) = \frac{1}{4\pi\varepsilon\varepsilon_0}\sum_{n,m=0}^{\infty}\left(\frac{\left(q_1^{(n)}+q_3^{(n)}-\delta_{n,0}\right)\left(q_2^{(m)}+q_4^{(m)}-\delta_{m,0}\right)}{\sqrt{x^2+(2nR)^2+(2mR)^2}}\right)(1-\delta_{n+m,0})$$

Image charges:

$$q_{\beta,n+1} = \kappa_\beta q_{\alpha,n}$$

$$\begin{cases} q_{a,n+1} = \kappa_a q_{d,n} & q_{d,n+1} = \kappa_d q_{a,n} \\ q_{c,m+1} = \kappa_c q_{b,m} & q_{b,m+1} = \kappa_b q_{c,m} \\ q_{\alpha,0} = 1 \leftarrow \forall\alpha = \{a,b,c,d\} \end{cases}$$

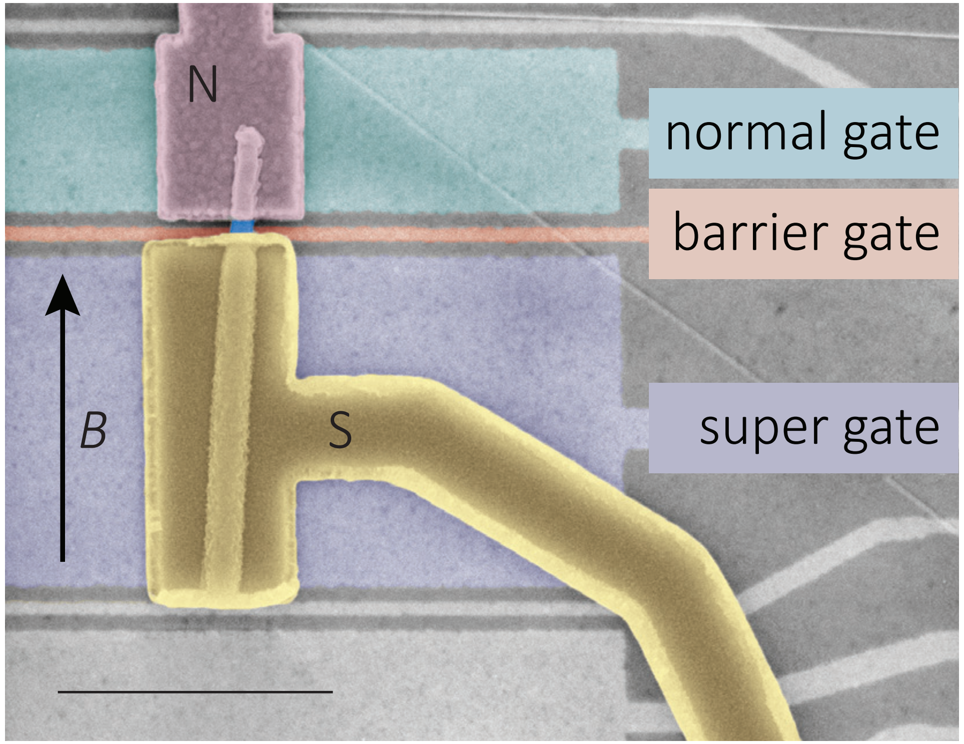
Interaction including the leads:

$$V_b(x) = \frac{1}{4\pi\varepsilon\varepsilon_0}\sum_{n,m,k=0}^{\infty}\left(\frac{\left(q_1^{(n)}+q_3^{(n)}-\delta_{n,0}\right)\left(q_2^{(m)}+q_4^{(m)}-\delta_{m,0}\right)q_{M_1}^{(k)}}{\sqrt{\left(x-(-1)^k\left(2^{\mathrm{floor}\left(\frac{k}{2}+1\right)}L-2L+x'\right)\right)^2+(2nR)^2+(2mR)^2}}+\right.$$
$$\left.+\frac{\left(q_1^{(n)}+q_3^{(n)}-\delta_{n,0}\right)\left(q_2^{(m)}+q_4^{(m)}-\delta_{m,0}\right)\left(q_{M_2}^{(k)}-\delta_{k,0}\right)}{\sqrt{\left(x+(-1)^k\left(2^{\mathrm{floor}\left(\frac{k+1}{2}\right)}L-x'\right)\right)^2+(2nR)^2+(2mR)^2}}\right)(1-\delta_{n+m+k,0})$$

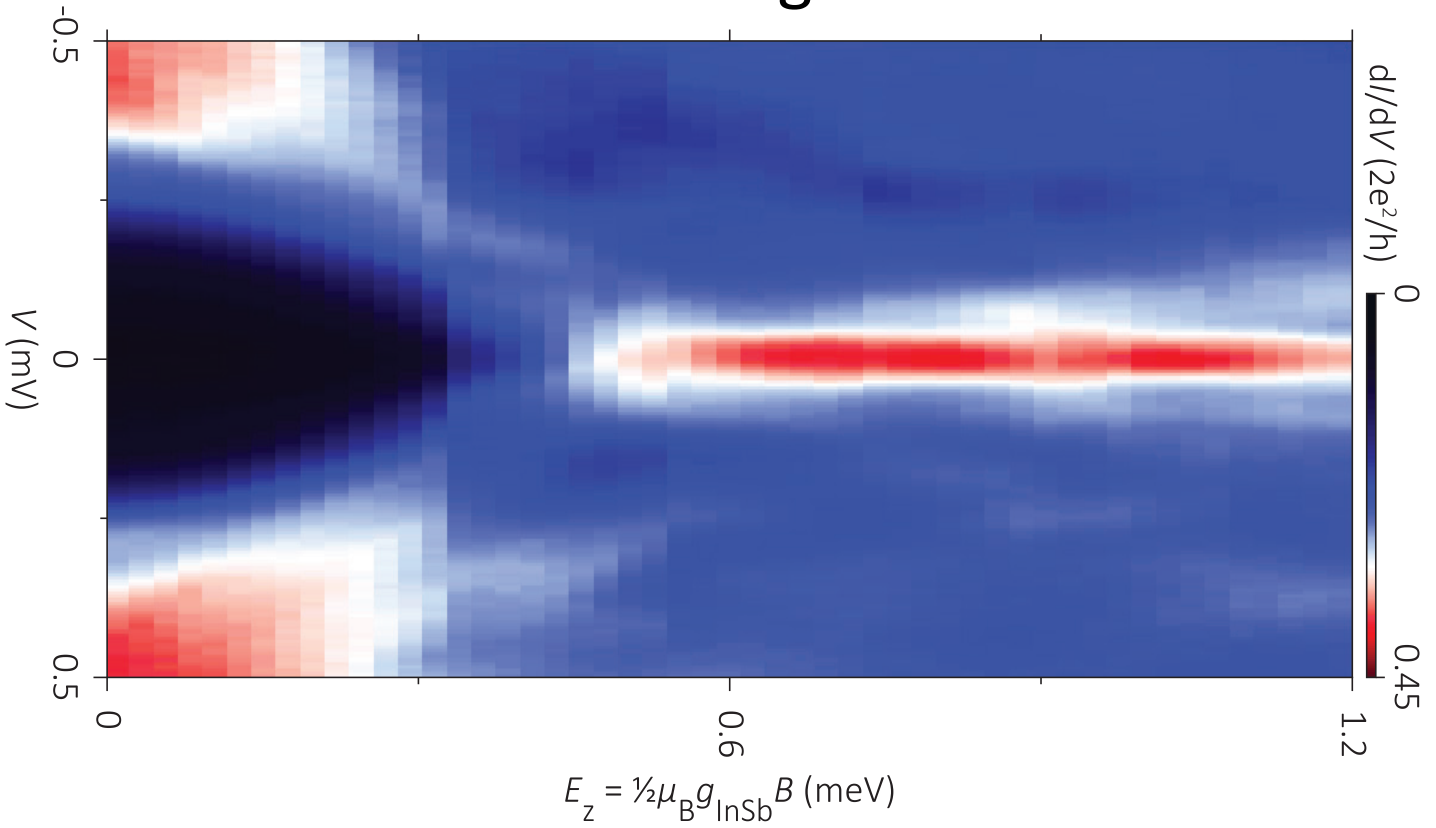
Ballistic Majorana nanowire devices

Hao Zhang,[1,2,*] Önder Gül,[1,2,*] Sonia Conesa-Boj,[1,2,3] Kun Zuo,[1,2] Vincent Mourik,[1,2]
Folkert K. de Vries,[1,2] Jasper van Veen,[1,2] David J. van Woerkom,[1,2] Michał P. Nowak,[1,2]
Michael Wimmer,[1,2] Diana Car,[3] Sébastien Plissard,[2,3] Erik P. A. M. Bakkers,[1,2,3] Marina Quintero-Pérez,[1,4]
Srijit Goswami,[1,2] Kenji Watanabe,[5] Takashi Taniguchi,[5] and Leo P. Kouwenhoven[1,2,†]

Experimental set up



Conductance through the nanowire

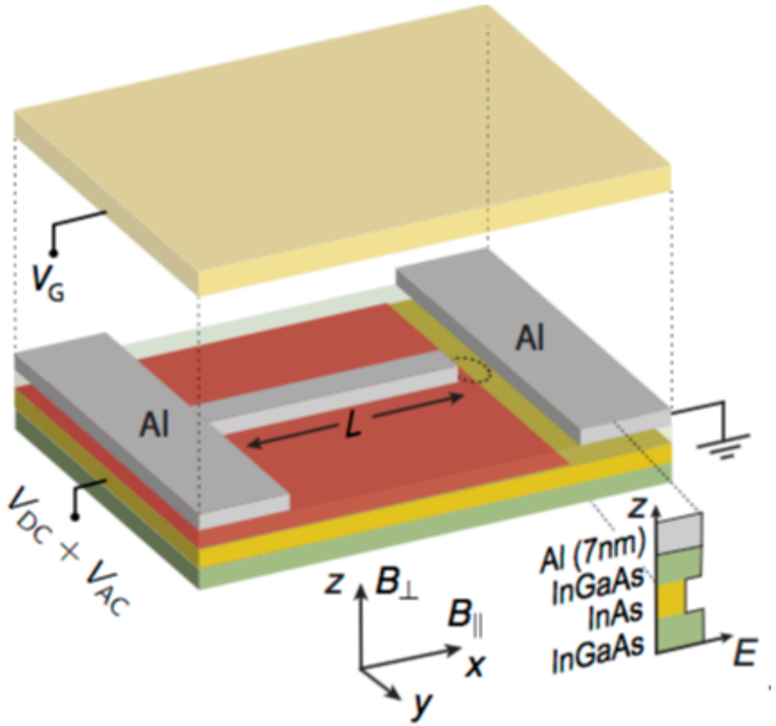


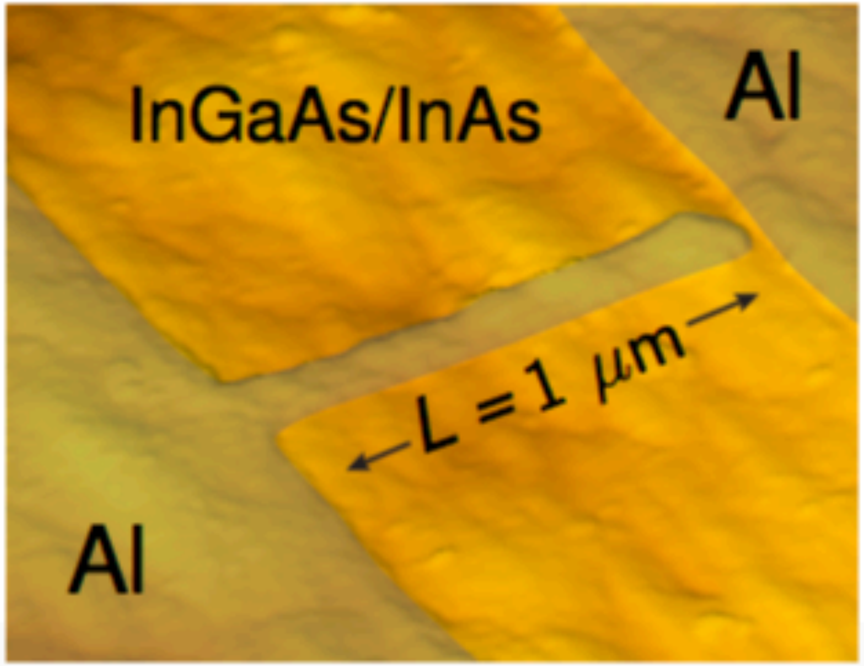
F. Experiments

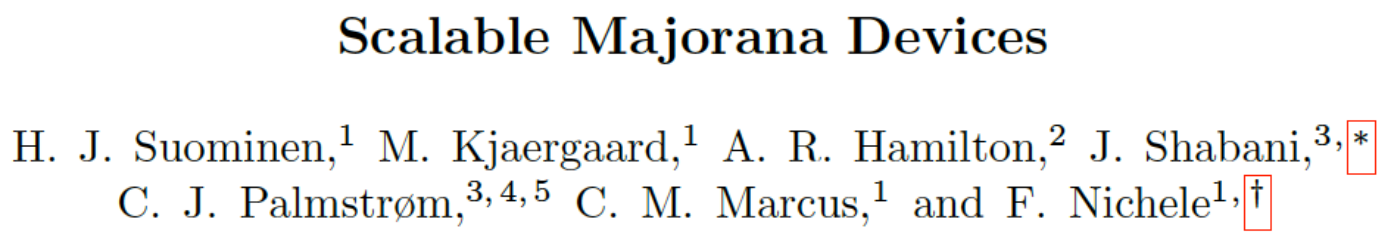
Scalable Majorana Devices

H. J. Suominen,[1] M. Kjaergaard,[1] A. R. Hamilton,[2] J. Shabani,[3,*]
C. J. Palmstrøm,[3,4,5] C. M. Marcus,[1] and F. Nichele[1,†]

Experimental set up







Conductance through the nanowire